Klausur NT 2024

1 a)

$$\max_{x_A \geq 0} \; 2\sqrt{x_A + x_B + x_C} - x_A$$

BEO

$$2 \cdot \frac{1}{2\sqrt{x_A + x_B + x_C}} - 1 = 0 \quad (\text{falls } x_A^* > 0)$$

$$\Leftrightarrow \frac{1}{\sqrt{x_A + x_B + x_C}} = 1 \Leftrightarrow \sqrt{x_A + x_B + x_C} = 1 \Rightarrow x_A + x_B + x_C = 1$$

$$\Rightarrow x_A = 1 - x_B - x_C \quad \text{falls} \quad x_B + x_C < 1$$

b) Beste Antwort von Adrian:

$$x_A^*(x_B, x_C) = \begin{cases} 1 - x_B - x_C, & \text{falls } x_B + x_C < 1 \\ 0, & \text{falls } x_B + x_C \geq 1 \end{cases}$$

c) (x_A^*, x_B^*, x_C^*) NGG, falls

$$\left.\begin{aligned} x_A^* &= x_A^*(x_B^*, x_C^*) \\ x_B^* &= x_B^*(x_A^*, x_C^*) \\ x_C^* &= x_C^*(x_A^*, x_B^*) \end{aligned}\right\} \Rightarrow \begin{aligned} &x_A^* + x_B^* + x_C^* = 1 \\ &x_A^*, x_B^*, x_C^* \geq 0 \end{aligned}$$

d) $x_A^* + x_B^* + x_C^* = 1$

e $x_i^* = x^* \qquad i = A, B, C$

$\Rightarrow \quad x_i^* = \frac{1}{3}$

$U_i(x_A^*, x_B^*, x_C^*) = 2\sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} - \frac{1}{3} = 2 - \frac{1}{3} = \frac{5}{3}$

e) $\max \quad 3 \cdot 2 \cdot \sqrt{x_A + x_B + x_C} - x_A - x_B - x_C$

$x_A, x_B, x_C \geq 0$

$\max \quad 6 \cdot \sqrt{x} - x$

$x \geq 0$

BEO (falls $x^{**} > 0$)

$6 \frac{1}{2\sqrt{x}} - 1 = 0 \iff \frac{3}{\sqrt{x}} = 1 \iff 3 = \sqrt{x} \Rightarrow x = 9$

$U_i(x_A^{**}, x_B^{**}, x_C^{**}) = 2 \cdot \sqrt{9} - x_i^{**} = 2 \cdot 3 - x_i^{**} = 6 - x_i^{**}$

Ist $(x_A^{**}, x_B^{**}, x_C^{**})$ NGG? Nein! $\sum_i x_i^{**} = 9 \neq 1$

Falls $x_i^{**} = 3 \Rightarrow U_i^{**} = 6 - 3 = 3 = \frac{9}{3} > \frac{5}{3}$