In[1]:=
```
T1 = 1/2 * {{0, 1, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
T2 = 1/2 * {{0, -I, 0, 0}, {I, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}
T3 = 1/2 * {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

T4 = 1/2 * {{0, 0, 1, 0}, {0, 0, 0, 0}, {1, 0, 0, 0}, {0, 0, 0, 0}}
T5 = 1/2 * {{0, 0, -I, 0}, {0, 0, 0, 0}, {I, 0, 0, 0}, {0, 0, 0, 0}}

T6 = 1/2 * {{0, 0, 0, 0}, {0, 0, 1, 0}, {0, 1, 0, 0}, {0, 0, 0, 0}}
T7 = 1/2 * {{0, 0, 0, 0}, {0, 0, -I, 0}, {0, I, 0, 0}, {0, 0, 0, 0}}
T8 = 1/(2 * Sqrt[3]) {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -2, 0}, {0, 0, 0, 0}}

T9 = 1/2 * {{0, 0, 0, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}, {1, 0, 0, 0}}
T10 = 1/2 * {{0, 0, 0, -I}, {0, 0, 0, 0}, {0, 0, 0, 0}, {I, 0, 0, 0}}

T11 = 1/2 * {{0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 0, 0}, {0, 1, 0, 0}}
T12 = 1/2 * {{0, 0, 0, 0}, {0, 0, 0, -I}, {0, 0, 0, 0}, {0, I, 0, 0}}

T13 = 1/2 * {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 1}, {0, 0, 1, 0}}
T14 = 1/2 * {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, -I}, {0, 0, I, 0}}

T15 = 1/(2 * Sqrt[6]) {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, -3}}


E1 = T1 + I * T2
Em1 = T1 - I * T2

E2 = T4 + I * T5
Em2 = T4 - I * T5

E3 = T6 + I * T7
Em3 = T6 - I * T7

E4 = T9 + I * T10
Em4 = T9 - I * T10

E5 = T11 + I * T12
Em5 = T11 - I * T12

E6 = T13 + I * T14
Em6 = T13 - I * T14

w1 = {T3[[1, 1]], T8[[1, 1]], T15[[1, 1]]}
w2 = {T3[[2, 2]], T8[[2, 2]], T15[[2, 2]]}
w3 = {T3[[3, 3]], T8[[3, 3]], T15[[3, 3]]}
```

```
w4 = {T3[[4, 4]], T8[[4, 4]], T15[[4, 4]]}

r1 = {a, b, c};
r1 = r1 /. Solve[(T3.E1 - E1.T3) == a * E1, a];
r1 = r1 /. Solve[(T8.E1 - E1.T8) == b * E1, b];
r1 = r1 /. Solve[(T15.E1 - E1.T15) == c * E1, c];
r1 = Flatten[r1]

r2 = {a, b, c};
r2 = r2 /. Solve[(T3.E2 - E2.T3) == a * E2, a];
r2 = r2 /. Solve[(T8.E2 - E2.T8) == b * E2, b];
r2 = r2 /. Solve[(T15.E2 - E2.T15) == c * E2, c];
r2 = Flatten[r2]

r3 = {a, b, c};
r3 = r3 /. Solve[(T3.E3 - E3.T3) == a * E3, a];
r3 = r3 /. Solve[(T8.E3 - E3.T8) == b * E3, b];
r3 = r3 /. Solve[(T15.E3 - E3.T15) == c * E3, c];
r3 = Flatten[r3]

r4 = {a, b, c};
r4 = r4 /. Solve[(T3.E4 - E4.T3) == a * E4, a];
r4 = r4 /. Solve[(T8.E4 - E4.T8) == b * E4, b];
r4 = r4 /. Solve[(T15.E4 - E4.T15) == c * E4, c];
r4 = Flatten[r4]

r5 = {a, b, c};
r5 = r5 /. Solve[(T3.E5 - E5.T3) == a * E5, a];
r5 = r5 /. Solve[(T8.E5 - E5.T8) == b * E5, b];
r5 = r5 /. Solve[(T15.E5 - E5.T15) == c * E5, c];
r5 = Flatten[r5]

r6 = {a, b, c};
r6 = r6 /. Solve[(T3.E6 - E6.T3) == a * E6, a];
r6 = r6 /. Solve[(T8.E6 - E6.T8) == b * E6, b];
r6 = r6 /. Solve[(T15.E6 - E6.T15) == c * E6, c];
r6 = Flatten[r6]


rm1 = {a, b, c};
rm1 = rm1 /. Solve[(T3.Em1 - Em1.T3) == a * Em1, a];
rm1 = rm1 /. Solve[(T8.Em1 - Em1.T8) == b * Em1, b];
rm1 = rm1 /. Solve[(T15.Em1 - Em1.T15) == c * Em1, c];
rm1 = Flatten[rm1]
```

```
rm2 = {a, b, c};
rm2 = rm2 /. Solve[(T3.Em2 - Em2.T3) == a * Em2, a];
rm2 = rm2 /. Solve[(T8.Em2 - Em2.T8) == b * Em2, b];
rm2 = rm2 /. Solve[(T15.Em2 - Em2.T15) == c * Em2, c];
rm2 = Flatten[rm2];

rm3 = {a, b, c};
rm3 = rm3 /. Solve[(T3.Em3 - Em3.T3) == a * Em3, a];
rm3 = rm3 /. Solve[(T8.Em3 - Em3.T8) == b * Em3, b];
rm3 = rm3 /. Solve[(T15.Em3 - Em3.T15) == c * Em3, c];
rm3 = Flatten[rm3]

rm4 = {a, b, c};
rm4 = rm4 /. Solve[(T3.Em4 - Em4.T3) == a * Em4, a];
rm4 = rm4 /. Solve[(T8.Em4 - Em4.T8) == b * Em4, b];
rm4 = rm4 /. Solve[(T15.Em4 - Em4.T15) == c * Em4, c];
rm4 = Flatten[rm4]

rm5 = {a, b, c};
rm5 = rm5 /. Solve[(T3.Em5 - Em5.T3) == a * Em5, a];
rm5 = rm5 /. Solve[(T8.Em5 - Em5.T8) == b * Em5, b];
rm5 = rm5 /. Solve[(T15.Em5 - Em5.T15) == c * Em5, c];
rm5 = Flatten[rm5]

rm6 = {a, b, c};
rm6 = rm6 /. Solve[(T3.Em6 - Em6.T3) == a * Em6, a];
rm6 = rm6 /. Solve[(T8.Em6 - Em6.T8) == b * Em6, b];
rm6 = rm6 /. Solve[(T15.Em6 - Em6.T15) == c * Em6, c];
rm6 = Flatten[rm6]

2 * (r1[[1]] * T3 + r1[[2]] * T8 + r1[[3]] * T15) === E1.Em1 - Em1.E1
2 * (r2[[1]] * T3 + r2[[2]] * T8 + r2[[3]] * T15) === E2.Em2 - Em2.E2
2 * (r3[[1]] * T3 + r3[[2]] * T8 + r3[[3]] * T15) === E3.Em3 - Em3.E3
2 * (r4[[1]] * T3 + r4[[2]] * T8 + r4[[3]] * T15) === E4.Em4 - Em4.E4
2 * (r5[[1]] * T3 + r5[[2]] * T8 + r5[[3]] * T15) === E5.Em5 - Em5.E5
2 * (r6[[1]] * T3 + r6[[2]] * T8 + r6[[3]] * T15) === E6.Em6 - Em6.E6
```

Out[1]= $\left\{\left\{0, \frac{1}{2}, 0, 0\right\}, \left\{\frac{1}{2}, 0, 0, 0\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\}$

Out[2]= $\left\{\left\{0, -\frac{i}{2}, 0, 0\right\}, \left\{\frac{i}{2}, 0, 0, 0\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\}$

Out[3]= $\left\{\left\{\frac{1}{2}, 0, 0, 0\right\}, \left\{0, -\frac{1}{2}, 0, 0\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\}$

Out[4]= $\left\{\left\{0, 0, \frac{1}{2}, 0\right\}, \{0, 0, 0, 0\}, \left\{\frac{1}{2}, 0, 0, 0\right\}, \{0, 0, 0, 0\}\right\}$

Out[5]= $\left\{\left\{0, 0, -\frac{i}{2}, 0\right\}, \{0, 0, 0, 0\}, \left\{\frac{i}{2}, 0, 0, 0\right\}, \{0, 0, 0, 0\}\right\}$

Out[6]= $\left\{\{0, 0, 0, 0\}, \left\{0, 0, \frac{1}{2}, 0\right\}, \left\{0, \frac{1}{2}, 0, 0\right\}, \{0, 0, 0, 0\}\right\}$

Out[7]= $\left\{\{0, 0, 0, 0\}, \left\{0, 0, -\frac{i}{2}, 0\right\}, \left\{0, \frac{i}{2}, 0, 0\right\}, \{0, 0, 0, 0\}\right\}$

Out[8]= $\left\{\left\{\frac{1}{2\sqrt{3}}, 0, 0, 0\right\}, \left\{0, \frac{1}{2\sqrt{3}}, 0, 0\right\}, \left\{0, 0, -\frac{1}{\sqrt{3}}, 0\right\}, \{0, 0, 0, 0\}\right\}$

Out[9]= $\left\{\left\{0, 0, 0, \frac{1}{2}\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{\frac{1}{2}, 0, 0, 0\right\}\right\}$

Out[10]= $\left\{\left\{0, 0, 0, -\frac{i}{2}\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{\frac{i}{2}, 0, 0, 0\right\}\right\}$

Out[11]= $\left\{\{0, 0, 0, 0\}, \left\{0, 0, 0, \frac{1}{2}\right\}, \{0, 0, 0, 0\}, \left\{0, \frac{1}{2}, 0, 0\right\}\right\}$

Out[12]= $\left\{\{0, 0, 0, 0\}, \left\{0, 0, 0, -\frac{i}{2}\right\}, \{0, 0, 0, 0\}, \left\{0, \frac{i}{2}, 0, 0\right\}\right\}$

Out[13]= $\left\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{0, 0, 0, \frac{1}{2}\right\}, \left\{0, 0, \frac{1}{2}, 0\right\}\right\}$

Out[14]= $\left\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \left\{0, 0, 0, -\frac{i}{2}\right\}, \left\{0, 0, \frac{i}{2}, 0\right\}\right\}$

Out[15]= $\left\{\left\{\frac{1}{2\sqrt{6}}, 0, 0, 0\right\}, \left\{0, \frac{1}{2\sqrt{6}}, 0, 0\right\}, \left\{0, 0, \frac{1}{2\sqrt{6}}, 0\right\}, \left\{0, 0, 0, -\frac{\sqrt{\frac{3}{2}}}{2}\right\}\right\}$

Out[16]= $\{\{0, 1, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[17]= $\{\{0, 0, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[18]= $\{\{0, 0, 1, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[19]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{1, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[20]= $\{\{0, 0, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[21]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[22]= $\{\{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[23]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{1, 0, 0, 0\}\}$

Out[24]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

Out[25]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 1, 0, 0\}\}$

Out[26]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 1\}, \{0, 0, 0, 0\}\}$

Out[27]= $\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 1, 0\}\}$

Out[28]= $\left\{\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right\}$

Out[29]= $\left\{-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right\}$

Out[30]= $\left\{0, -\frac{1}{\sqrt{3}}, \frac{1}{2\sqrt{6}}\right\}$

Out[31]= $\left\{0, 0, -\frac{\sqrt{\frac{3}{2}}}{2}\right\}$

Out[36]= $\{1, 0, 0\}$

Out[41]= $\left\{\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right\}$

Out[46]= $\left\{-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right\}$

Out[51]= $\left\{\frac{1}{2}, \frac{1}{2\sqrt{3}}, \sqrt{\frac{2}{3}}\right\}$

Out[56]= $\left\{-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \sqrt{\frac{2}{3}}\right\}$

Out[61]= $\left\{0, -\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right\}$

Out[66]= $\{-1, 0, 0\}$

Out[76]= $\left\{\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right\}$

Out[81]= $\left\{-\frac{1}{2}, -\frac{1}{2\sqrt{3}}, -\sqrt{\frac{2}{3}}\right\}$

Out[86]= $\left\{\frac{1}{2}, -\frac{1}{2\sqrt{3}}, -\sqrt{\frac{2}{3}}\right\}$

Out[91]= $\left\{0, \frac{1}{\sqrt{3}}, -\sqrt{\frac{2}{3}}\right\}$

Out[92]= True

Out[93]= True

Out[94]= True

Out[95]= True

Out[96]= True

Out[97]= True