Lorentz-Transformationen

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$$\begin{pmatrix} \Lambda^0{}_0 & \Lambda^0{}_1 & \Lambda^0{}_2 \\ \Lambda^1{}_0 & \Lambda^1{}_1 & \Lambda^1{}_2 \\ \Lambda^2{}_0 & \Lambda^2{}_1 & \Lambda^2{}_2 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \end{pmatrix} = \begin{pmatrix} x'^0 \\ x'^1 \\ x'^2 \end{pmatrix}$$

E_2 und E_3 in die x'^2 Gleichung

$$\Lambda^2{}_0 \frac{lc}{c-v} + \Lambda^2{}_1 \frac{lc}{c-v} = 0 \tag{1}$$

$$\Lambda^2{}_0 \frac{2lc^2}{c-v} + \Lambda^2{}_1 \frac{2lcv}{c-v} = 0 \tag{2}$$

damit ist $\Lambda^2{}_0 = 0$ und $\Lambda^2{}_1 = 0$.

$$\begin{pmatrix} \Lambda^0{}_0 & \Lambda^0{}_1 & \Lambda^0{}_2 \\ \Lambda^1{}_0 & \Lambda^1{}_1 & \Lambda^1{}_2 \\ \color{red}{0} & \color{red}{0} & \Lambda^2{}_2 \end{pmatrix}$$

E_2 und E_3 in die x'^1 Gleichung

$$\Lambda^1{}_0 \frac{lc}{c-v} + \Lambda^1{}_1 \frac{lc}{c-v} = l' \tag{3}$$

$$\Lambda^1{}_0 \frac{2lc^2}{c-v} + \Lambda^1{}_1 \frac{2lcv}{c-v} = 0 \tag{4}$$

Definiere

$$\gamma := \frac{l'}{l} = \Lambda^1{}_0 \frac{c}{c-v} + \Lambda^1{}_1 \frac{c}{c-v}$$

damit wird $\Lambda^1{}_0 = -\frac{v}{c}\gamma$ und $\Lambda^1{}_1 = \gamma$.

$$\begin{pmatrix} \Lambda^0{}_0 & \Lambda^0{}_1 & \Lambda^0{}_2 \\ -\frac{v}{c}\gamma & \gamma & \Lambda^1{}_2 \\ 0 & 0 & \Lambda^2{}_2 \end{pmatrix}$$

E_2 und E_3 in die x'^0 Gleichung

$$\Lambda^0{}_0 \frac{lc}{c-v} + \Lambda^0{}_1 \frac{lc}{c-v} = l' \qquad (5)$$

$$\Lambda^0{}_0 \frac{2lc^2}{c^2-v^2} + \Lambda^0{}_1 \frac{2lcv}{c^2-v^2} = 2l' \qquad (6)$$

Aus diesen beiden Gleichungen ergibt sich $\Lambda^0{}_1 = -\frac{v}{c}\gamma$ und $\Lambda^0{}_0 = \gamma$.

$$\begin{pmatrix} \gamma & -\frac{v}{c}\gamma & \Lambda^0{}_2 \\ -\frac{v}{c}\gamma & \gamma & \Lambda^1{}_2 \\ 0 & 0 & \Lambda^2{}_2 \end{pmatrix}$$

E_4 in die x'^1 Gleichung

$$-\frac{v}{c}\gamma\frac{c\tilde{l}}{\sqrt{c^2-v^2}}+\gamma\frac{v\tilde{l}}{\sqrt{c^2-v^2}}+\Lambda^1{}_2\tilde{l}=0 \tag{7}$$

$\Lambda^1{}_2 = 0.$

$$\begin{pmatrix} \gamma & -\frac{v}{c}\gamma & \Lambda^0{}_2 \\ -\frac{v}{c}\gamma & \gamma & 0 \\ 0 & 0 & \Lambda^2{}_2 \end{pmatrix}$$

E_5 in die x'^0 Gleichung

$$\gamma \frac{2\tilde{l}c}{\sqrt{c^2 - v^2}} - \frac{v}{c}\gamma \frac{2v\tilde{l}}{\sqrt{c^2 - v^2}} = 2l' \tag{8}$$

Definiere

$$\alpha := \frac{l'}{\tilde{l}} = \frac{\gamma c}{\sqrt{c^2 - v^2}} - \frac{v}{c}\gamma \frac{v}{\sqrt{c^2 - v^2}} = \gamma \sqrt{1 - \frac{v^2}{c^2}}$$

Somit

$$\gamma = \frac{\alpha}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

E_4 in die x'^0 Gleichung

$$\gamma \frac{c\tilde{l}}{\sqrt{c^2 - v^2}} - \frac{v}{c}\gamma \frac{v\tilde{l}}{\sqrt{c^2 - v^2}} + \Lambda^0{}_2 \tilde{l} = l' \tag{9}$$

ergibt:

$$\gamma \sqrt{1 - \frac{v^2}{c^2}} + \Lambda^0{}_2 = \alpha + \Lambda^0{}_2 = \alpha \tag{10}$$

und somit $\Lambda^0{}_2 = 0$.

$$\begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 \\ 0 & 0 & \Lambda^2{}_2 \end{pmatrix}$$

E_4 in die x'^2 Gleichung

$$\Lambda^2{}_2 = \frac{l'}{\tilde{l}} = \alpha \tag{11}$$

ergibt:

$$\begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 \\ 0 & 0 & \alpha \end{pmatrix}$$

Skalentransformation:

$$x'^{\mu} = \alpha x^{\mu}.$$

Betrachte inverse Lorentz-Transformation

$$\Lambda^{-1}(v) = \Lambda(-v)$$

also:

$$\begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 \\ 0 & 0 & \alpha \end{pmatrix} \begin{pmatrix} \gamma & \frac{v}{c}\gamma & 0 \\ \frac{v}{c}\gamma & \gamma & 0 \\ 0 & 0 & \alpha \end{pmatrix} = \begin{pmatrix} \alpha^2 & 0 & 0 \\ 0 & \alpha^2 & 0 \\ 0 & 0 & \alpha^2 \end{pmatrix}$$

damit $\alpha = 1$.