Problem Set 5

Labour Economics, Winter Semester 2025/26

Submit by Monday, 2 February, 22:45h ***on Moodle!***

Learning objectives

- Interpreting stylised empirical facts of shifting labour quantities and prices.
- Analysing CES-based and tasks-based models of technological changes.
- Applying these models to hypothesized future structural shifts.
- Familiarising with widely-used variance decompositions.

Tasks

A. The Canonical Model

Suppose from a CES production function with H_t units of input of high-school labor and C_t of college

$$y_t = \left[\theta_{ht} H_t^{\frac{\sigma-1}{\sigma}} + \theta_{ct} C_t^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

and competitive factor markets, we derive an estimation equation of the form:

$$\log\left(\frac{w_t^c}{w_t^h}\right) = \frac{\sigma-1}{\sigma} D_t - \frac{1}{\sigma} log(\frac{C_t}{H_t})$$

1. What are the parameters of interest and how do you interpret them?

   Solution

   a) $D_t = log(\frac{\theta_{ct}}{\theta_{ht}})$ is a relative demand index of shifts favouring college equivalents. Changes in D arise from skill-biased technical change D.
   b) The impact of changes in relative skill supplies $(\frac{C_t}{H_t})$ depends on the elasticity of substitution σ.

2. Are these parameters identified as it stands or which further restriction is necessary?

   Solution

a) flexible D_t is perfectly collinear with $log(\frac{C_t}{H_t})$

b) replacing D with a linear time trend ('$trend$'), $\log\left(\frac{w_t^c}{w_t^h}\right) = \gamma_0 + \gamma_1 trend + \gamma_2 log(\frac{C_t}{H_t}) + \epsilon_t$

3. Suppose we obtain a coefficient of circa $0.5 - 0.66$ on the regressor $log(\frac{C_t}{H_t})$. What does this imply for the model parameter(s). Are C and H complements or substitutes?

Solution

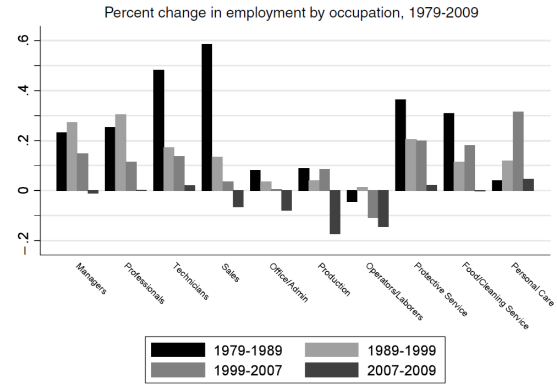
- Implies $\sigma \in [1.5, 2]$ and C and H are substitutes in production.
- Elasticity of substitution $\sigma = \frac{d\ln(C/H)}{d\ln(w^h/w^c)}\Big|_{Y constant}$

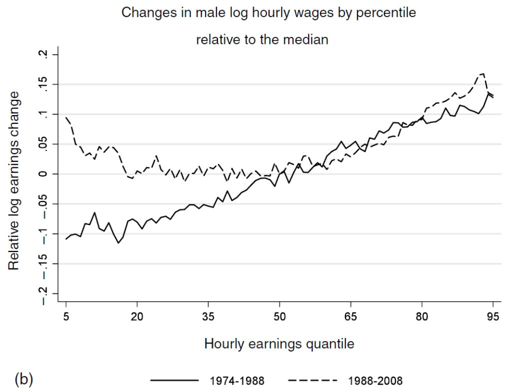
4. In empirical studies, which of the factors has been more important in driving the college premium over the past 30 years?

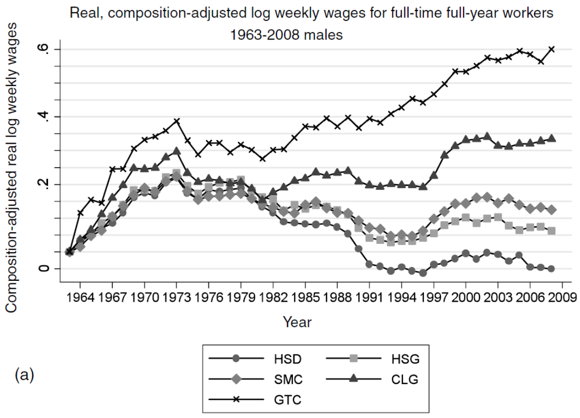
Solution

Demand outpaced supply of $(\frac{C_t}{H_t})$ increases up to circa 2010.

5. What are empirical problems with the canonical model of the race between the supply and demand for skill / education?







(a)

Solution

a) Job polarization
b) Wage polarization
c) Wages of some skill groups decline in absolute terms
d) Worse empirical fit of Katz-Murphy model from the mid-1990s onward

B. The Ricardian Model

1. Acemoglu and Autor (2011) propose an alternative task model, where each task can be produced by each of three skill groups:

$$y(i) = A_L \alpha_L(i) l(i) + A_M \alpha_M(i) m(i) + A_H \alpha_H(i) h(i)$$

Interpret this key equation and its components.

Solution

a) indices L, M, H refer to low, medium and high skill worker types
b) A_L, A_M, A_H: factor-augmenting technologie
c) $\alpha_L(i)$, $\alpha_M(i)$, $\alpha_H(i)$: productivity of worker types in task i
d) $l(i), m(i), h(i)$: number of workers allocated to task i

2. What are skills and what are tasks? Why is the distinction between them important and a key new feature of the task model?

Solution

- Skills refer to a worker's endowment of capabilities, such as education, experience, or cognitive ability.
- Tasks are units of work that need to be performed in the production process.

This distinction is important because different workers can perform the same tasks, and task allocation influences wage inequality. Wage differences arise when technology or supply shifts alter which tasks are assigned to different skill groups.

3. In what way do Acemoglu and Autor assume comparative advantage here?

Solution

a) $\alpha_L(i)/\alpha_M(i)$ and $\alpha_M(i)/\alpha_H(i)$ are differentiable and strictly decreasing
b) Higher indices of i correspond to more complex tasks
c) Comparative advantage as a function of a task's complexity

4. Can you sketch the properties of the model's equilibrium? Is there anything more to determine than prices? Do workers of the same skill earn different wages when they work in different tasks?

Solution

a) Endogenous allocation of skills to tasks. Two cutoffs I_L and I_H.
b) No. Law of one price as workers can costlessly reallocate.

Given the law of one price, prices (p(i): price of task (i)) solve from

$$\begin{aligned} w_L &= p(i) A_L \alpha_L(i) \quad \text{for any} \quad i < I_L \\ w_M &= p(i) A_M \alpha_M(i) \quad \text{for any} \quad I_L < i < I_H \\ w_H &= p(i) A_H \alpha_H(i) \quad \text{for any} \quad i > I_H. \end{aligned}$$

This gives, e.g. $p(i)\alpha_L(i) = p(i')\alpha_L(i') = P_L$: price index of tasks performed by low skill workers

5. Suppose a recent study showed that high-skilled workers are "most affected" by artificial intelligence. How would you analyze this in this model? Is what you are suggesting skill-complementing or task-replacing technological change?

Solution

Suppose that AI raises A_H: $A_H \uparrow \rightarrow I_H \downarrow; I_L \downarrow; (I_H - I_L) \downarrow$

- A rise in A_H makes high skill workers uniformly more productive
- Number of tasks in which H hold comparative advantage over M increases
- Consequently, tasks get shifted away from M to H workers
- If I_L remained constant, excess supply of M workers
- Therefore, *indirect effect*: reduce I_L to shift tasks from L to M
- Note: Direct effect always dominates indirect effect

As in the canonical model, increase in A_H raises both $\frac{w_H}{w_L}$ and $\frac{w_H}{w_M}$

- Importantly, A_H unambiguously reduces $\frac{w_M}{w_L}$

- Direct effect of $\uparrow A_H$: reallocation of tasks from M to H workers
- Indirect effect: reallocation of jobs from L to M workers
- Yet, direct effect > indirect effect, such that $\frac{w_M}{w_L} < 0$

6. Alternatively, assume "most affected" means that artificial intelligence carries out the top most complex (highest-ranked) tasks at almost zero cost.

$$y(i) = A_L\alpha_L(i)l(i) + A_M\alpha_M(i)m(i) + A_H\alpha_H(i)h(i) + A_K\alpha_K(i)k(i)$$

Productivity $\alpha_K(i)$ increases such over time that now $[I_K, 1] \in [I_H, 1]$ are profitably produced by capital.

Solution

Leads to new equilibrium with new threshold tasks $\hat{I}_L$ and $\hat{I}_H$ s.t.

- Threshold ranks: $0 < \hat{I}_L < \hat{I}_H < I_K < 1$
- Skill allocations:
  - for any $i < \hat{I}_L$ we have $l(i) = L/\hat{I}_L$
  - for any $i \in (\hat{I}_L, \hat{I}_H)$ we have $m(i) = \frac{M}{\hat{I}_H - \hat{I}_L}$
  - for any $i \in (\hat{I}_H, I_K)$ we have $h(i) = H/(I_K - \hat{I}_H)$
- Relative wages: $\frac{w_M}{w_H} \uparrow$, $\frac{w_L}{w_M} \uparrow$.

  In words, change in the productivity of capital leads to:
  - job downgrading (in terms of complexity and for all workers)
  - strongest for H and weakest for L workers as direct effect > indirect effect.
  - wage inequality declines across the board.

7. Big picture, how does the Ricardian model improve upon the canonical model of skill-biased technical change in giving a fuller explanation of the impact of technological changes?

Solution

The canonical model assumes two skill groups (high and low) and focuses on how technology increases demand for skilled labor, leading to wage inequality.

The Ricardian model distinguishes between skills and tasks, allowing for task reallocation across skill groups.

- This explains job quantity changes (e.g. polarization in the past), where middle-skill jobs decline due to automation while low- and high-skill jobs grow.
- Task allocation influences wage inequality. Wage differences arise when the tasks that are assigned to different skill groups change.

C. Decomposition Approaches

Finally, influential papers have studied firm-level wage premia, sorting and between-versus-within variances of log wages.

1. Card et al. 2013 decompose wage variation from the AKM model

$$w_{it} = \alpha_i + \psi_{J(i,t)} + X_{it}'\beta + \epsilon_{it} \tag{1}$$

where w_{it} is the log wage of worker i at time t, X_{it} the workers' observable and time-varying characteristics (e.g. experience), and $J(i,t)$ the firm that he/she works at in time t. Can you decompose wage inequality according to this model into the variances and covariances of its constituent parts? Which of these are commonly found to be the more important [*it is sufficient to have some very rough idea of how large they are*]?

Solution

Interpret the components of this estimation model, which is known as "AKM":

α_i: the person effect which captures the unobserved heterogeneity in workers ability.
$\psi_{J(i,t)}$: the firm-specific relative wage premium.
$X_{it}'\beta$: time-varying index controlling for observable heterogeneity in workers characteristics.
ϵ_{it}: unobserved time-varying error, which may contain shocks to human capital, person-specific jobmatch effects, and other factors.

Can you decompose wage inequality according to this model into the variances and covariances of its constituent parts?

$$\begin{aligned} Var(w_{iJt}) &= Var(\alpha_i) + Var(\psi_{J(i,t)}) + Var(X_{it}'\beta) + Var(\epsilon_{it}) \\ &+2Cov(\alpha_i, \psi_{J(i,t)}) + 2Cov(\alpha_i, X_{it}'\beta) \\ &+2Cov(\psi_{J(i,t)}, X_{it}'\beta) \end{aligned}$$

Which of these are commonly found to be the more important?

- The component that explains the largest proportion of the wage variation is the variation in person effect, which explains more than 50% of the wage variation.
- The variation of firm-specific effect and the covariance of person and firm effect explains about 20% of the wage variation, respectively.
- Though not being the most important factors, the variation of firm-specific effect and the covariance of person and firm effect are not as important as the variance of person effect in terms of explaining the wage dispersion, we are quite interested in these two measures because the former indicates the importance of firm heterogeneity in determining the wage difference and the later suggests the sorting effect also contributes to the wage inequality.

2. Song et al. (2019) take one step back and first decompose w_{iJ} (we drop the t index here for convenience) into a within and a between component:

$$var(w_{iJ}) = var(w_J) + var(e_{iJ})$$

Here w_J is the firm average while $e_{iJ} \equiv w_{iJ} - w_J$ is the individual deviation from that average. Can you prove that this decomposition is correct?

Solution

By construction, deviations have zero mean within firms,

$$\mathbb{E}[e_{iJ} \mid J] = 0,$$

which implies $\mathbb{E}[e_{iJ} \mid w_J] = 0$ and then $\text{Cov}(w_J, e_{iJ}) = 0$.

Using the variance of a sum,

$$\begin{aligned} \text{Var}(w_{iJ}) &= \text{Var}(w_J + e_{iJ}) \\ &= \text{Var}(w_J) + \text{Var}(e_{iJ}) + 2\,\text{Cov}(w_J, e_{iJ}) \\ &= \text{Var}(w_J) + \text{Var}(e_{iJ}). \end{aligned}$$

Thus, total wage dispersion decomposes into between-firm and within-firm parts only.

References

- Acemoglu/Autor (2011): Skills, Tasks and Technologies: Implications for Employment and Earnings, Handbook of Labor Economics (4b).
- Card/Kline/Heining (2013): Workplace Heterogeneity And The Rise Of The West German Wage Inequality, QJE.
- Song, J., Price, D. J., Guvenen, F., Bloom, N., and Von Wachter, T. (2019): Firming up inequality. The Quarterly journal of economics, 134(1), 1-50.