

Lecture 3c: Demand and Supply with Firm Heterogeneity

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Empirical Economics

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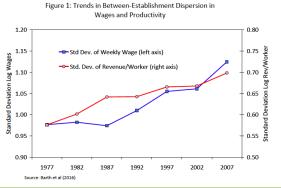
5a Labour Demand of Heterogeneous Firms

Based on "Firms and labor market inequality: Evidence and some theory" by David Card, Ana-Rute Cardoso, Jörg Heining, and Patrick Kline, 2018, Journal of Labor Economics.

- 1. Evidence:
 - a. Productivity, wages, and rent sharing.
 - b. Firm switching.
- 2. Theory:
 - a. Baseline model.
 - b. Extensions.

A large empirical literature documents that firms, like workers, exhibit vast heterogeneity in productivity (Syverson, 2011).

 It is natural to wonder if these large productivity differences lead to differences in worker pay.



Measuring rents:

 Suppose that there is a single type of labor at a firm j, and wage (w_i) is determined by a structural relationship of the form:

$$w_j = b + \gamma \frac{Q_j}{N_j} \tag{1}$$

where *b* represents an alternative wage, N_j is employment at the firm, Q_j represents quasi-rents, and γ is a rent-sharing parameter.

• Think of Q_j as a rent that can be distributed to the firm owners (profit) and to the worker (wage premium).

• The firm combines labor inputs and capital (K_j) and faces an exogenous rental rate r on capital, yielding the quasi-rent:

$$Q_j = VA_j - bN_j - rK_j = P_jT_jf(N_j, K_j) - bN_j - rK_j$$

where VA_j is value added (revenue net of materials costs), P_j is a potentially firm-specific selling price index, T_j is an index of technical efficiency, and f is a standard production function.

• Here P_jT_j represents total factor productivity (TFP_j) which is also referred to as "revenue productivity" because it is the product of "physical productivity" T_j and product price P_j .

 The elasticity of wages with respect to an exogenous change in quasi-rent per worker is:

$$\xi_{Qj} = \frac{\partial \ln w}{\partial \ln(\frac{Q_j}{N_j})} = \frac{\gamma \frac{Q_j}{N_j}}{b + \gamma \frac{Q_j}{N_j}}$$
(2)

 Under the usual bargaining interpretation of equation (1) profits per worker are a constant share of quasi-rents per worker:

$$\frac{\pi_j}{N_j} = (1 - \gamma) \frac{Q_j}{N_j}$$

The elasticity of wages with respect to value added per worker:

$$\xi_j = \xi_{Qj} imes rac{VA_j}{Q_j}$$

An important confounding factor in the rent sharing literature is variation in worker quality.

- Firms that employ more highly skilled workers would be expected to have higher revenue per worker and value added per worker, and also pay higher wages, leading to a potential upward bias in the measured rent sharing elasticity in cross-sectional studies that compare different firms at a point in time.
- A similar bias can also arise in longitudinal studies that compare changes in firm-specific wages and profitability over time if there are unobserved changes in the skill characteristics of workers.

Few studies have exogenous sources of variation in productivity.

- Exception is Van Reenen (1996), who studies the effects of major firm innovations (improved product or process) on employee wages and finds large elasticity of 0.29.
 - may be upward biased by skill upgrading on the part of innovative firms
- However, Kline, Petkova, Williams, and Zidar (2019) find equally large elasticity (0.29) when exploiting ex ante similar patents that were initially allowed versus rejected.
- With employer-employee micro data, economists have had to contend with serious measurement error problems that emerge when constructing plant-level productivity measures.

Example in Portugese Employer-Employee Data

Cross-Sectional and Within-Job Models of Rent Sharing for Portuguese

	Basic Specification (1)	Basic + Major Industry/City (2)	Basic + Detailed Industry/City (3)
A. Cross-sectional models (worker- year observations, 2005–9):			
OLS: rent measure = mean log value			
added per worker, 2005–9	.270	.241	.207
,	(.017)	(.015)	(.011)
OLS: rent measure = mean log sales	, ,	,	,
per worker, 2005–9	.153	.171	.159
	(.009)	(.007)	(.004)
IV: rent measure = mean log value added per worker, 2005–9; instrument = mean log sales per	, ,	` ,	, ,
worker, 2004–10	.327 (.014)	.324 (.011)	.292 (.008)
First-stage coefficient	.475 (t = 26.19)	.541	.562 (t = 64.38)

Example (continued)

B. Within-job models (change in wages			
from 2005 to 2009 for stayers):			
OLS: rent measure = change in log			
value added per worker from 2005			
to 2009	.041	.039	.034
	(.006)	(.005)	(.003)
OLS: rent measure = change in log			
sales per worker from 2005 to 2009	.015	.014	.013
	(.005)	(.004)	(.003)
IV: rent measure = change in log			
value added per worker from 2005			
to 2009; instrument = change in			
log sales per worker, 2004–10	.061	.059	.056
	(.018)	(.017)	(.016)
First-stage coefficient	.221	.217	.209
	(t = 11.82)	(t = 13.98)	(t = 18.63)

Note.—The sample in panel A is 2,503,336 person-year observations from Quadros de Pessoal (QP) for males working in 2005-9 between the ages of 19 and 65 years with at least 2 years of potential experience employed at a firm with complete value-added data (from Sistema de Analisis de Balances Ibericos [SABII) for 2005-9 and sales data (from QP) for 2004 and 2010. The sample in panel B is 284,071 males ages 19-61 years in 2005 who worked every year from 2005 to 2009 at a firm with complete value-added data (from SABI) for 2005-9 and sales data (from QP) for 2004 and 2010. Standard errors are clustered by firm (62,4845 firms in panel B). Models in panel A control for cubic in experience and unrestricted education × year dummies. Models in panel B control for a quadratic in experience and education. Models in col. 2 also control for 20 major industries and two major cities (Lisbon and Porto). Models in col. 3 also control for 202 detailed industry dummies and 29 Nomenclature of Territorial Units for Statistics region 3 location dummies. IV = instrumental variables; OLS = ordinary least squares.

Abowd, Kramarz, Margolis (1999, AKM) models:

AKM model for the log wage of person i in year t takes the form:

$$\log(\mathbf{w}_{it}) = \alpha_i + \psi_{J(i,t)} + \mathbf{X}'_{it}\beta + \epsilon_{it}$$

where

- $-X_{it}$ is a vector of time varying controls,
- $-\alpha_i$ is a "person effect" capturing the (time-invariant) portable component of earnings ability,
- the $\{\psi_j\}_{j=1}^J$ are firm-specific relative pay premiums, J(i,t) is a function indicating the employer of worker i in year t,
- and ϵ_{it} is an unobserved time-varying error capturing shocks to human capital, person-specific job match effects, and other factors.

Event study of pre&post trends, and symmetry of switching gains:

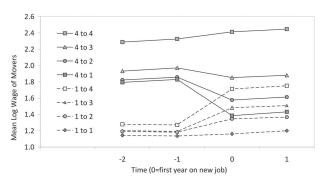


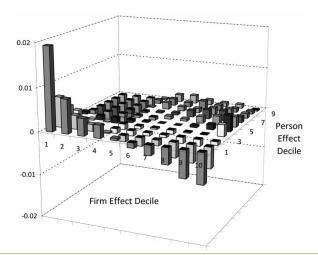
Fig. 3.—Mean log wages of Portuguese male job changers classified by quartile of coworker wages at origin and destination. The figure shows mean wages of male workers at mixed-gender firms who changed jobs in 2004–7 and held the preceding job for 2 years or more and the new job for 2 years or more. Jobs are classified into quartiles based on mean log wage of coworkers of both genders. Source: Card et al. (2016, fig. I). A color version of this figure is available online.

Main empirical issues in AKM:

- 1. Exogenous mobility, i.e., need worker switching uncorrelated with time-varying wage residual ϵ_{it} . Evidence in previous slide.
- 2. Since the coefficients are identified from wage changes when workers change firms, limited switching leads to measurement error and downward-bias in $cov(\alpha_i, \psi_i)$
- **3.** Additive separable functional form. Plot residuals across worker α_i and firm ψ_J effect deciles.
- 4. Some restriction to separate age from time effects.

Residuals across worker-firm deciles:

High averages at bottom left may reflect impact of minimum wage.



Presence of the firm effects allows for the possibility that some firms pay systematically higher or lower wages than other firms.

- Estimates of AKM style models in different countries have found that the firm effects in these models typically explain 15–25 percent of the variance of wages (5–15% in newer research).
- Less than the person effects but enough to indicate that firmspecific wage setting is important for wage inequality.
- If different firms pay different wage premiums, the pattern of sorting of workers to firms will also matter for overall wage inequality.

$$Var(\log(w_{it})) = Var(\alpha_i) + Var(\psi_{J(i,t)}) + Var(X'_{it}\beta) + Var(\epsilon_{it}) + 2Cov(\alpha_i, \psi_{J(i,t)}) + 2Cov(\alpha_i, X'_{it}\beta) + 2Cov(\psi_{J(i,t)}, X'_{it}\beta)$$

- AKM showed that the estimated firm-specific wage premiums were positively correlated with measures of firm profitability including value added per worker and sales per worker.
- The rent sharing elasticity obtained from a regression of wages on a time-invariant measure of rents at the current employer (γ_w) can be decomposed into the sum of three components reflecting the regression on firm-specific rents of the estimated worker effects (γ_α) , the estimated firm effects (γ_ψ) , and the time-varying covariate index $(\gamma_{X\beta})$.

$$\gamma_{\mathbf{W}} = \gamma_{\alpha} + \gamma_{\psi} + \gamma_{\mathbf{X}\beta}$$

The regression coefficients γ_{α} and $\gamma_{X\beta}$ represent sorting effects and γ_{ψ} is arguably a clean measure of the rent sharing elasticity.

Example in Portugese Employer-Employee Data

Relationship between Components of Wages and Mean Log Value

	Basic Specification (1)	Basic + Major Industry/City (2)	Basic + Detailed Industry/City (3)
A. Combined sample ($n = 2,252,436$ personyear observations at 41,120 firms):			
Log hourly wage	.250	.222	.187
	(.018)	(.016)	(.012)
Estimated person effect	.107	.093	.074
	(.010)	(.009)	(.006)
Estimated firm effect	.137	.123	.107
	(.011)	(.009)	(.008)
Estimated covariate index	.001	.001	.001
	(.000)	(.000)	(.000)
B. Less educated workers ($n = 1,674,676$ person-year observations at 36,179 firms)	:		
Log hourly wage	.239	.211	.181
	(.017)	(.016)	(.011)
Estimated person effect	.089	.072	.069
	(.009)	(.009)	(.005)
Estimated firm effect	.144	.133	.107
	(.015)	(.013)	(.008)
Estimated covariate index	.000	.000	.000

Example (continued)

C. More educated workers (n = 577,760

-year observations at 17,615 firm	1S):		
Log hourly wage	.275	.247	.196
	(.024)	(.020)	(.017)
Estimated person effect	.137	.130	.094
*	(.016)	(.013)	(.009)
Estimated firm effect	.131	.113	.099
	(.012)	(.009)	(.010)
Estimated covariate index	001	001	001
	(.000)	(.000)	(.000)
ed firm effect	.137 (.016) .131 (.012) 001	.130 (.013) .113 (.009) 001	0. 0.) 0. 0.)

NOTES.—Entries are coefficients of mean log value added per worker (at current firm) in regression models with dependent variables listed in the row headings. Standard errors are clustered by firm (in parentheses). The sample in panel B includes males with less than completed secondary education at firms in the connected set for less educated workers. The sample in panel C includes males with a high school education or more at Irms in the connected set for more educated workers. The sample in panel A includes males in either the panel B or the panel C sample. All models control for cubic in experience and unrestricted education x year dummies. Models in col. 2 also control for 20 major industries and two major cities (Lisbon and Porto). Models in col. 3 also control for 20 agoin ridustries and two major cities (Lisbon and Porto). Models in col. 3 also control for 20 detailed industries and two major cities (Lisbon and Porto). Both since the panel of the parent part of the parent pa

- The rent sharing elasticities estimated from a cross-sectional specification incorporate a sizable worker quality bias.
- In each column of the table, roughly 40% of the overall wage elasticity in row 1 is due to the correlation of worker quality (measured by the person effect component of wages) with firm specific quality.
- Adjusting for worker quality, the estimates in row 3 point to a rent sharing elasticity in the range of 0.10 to 0.15, large enough to create a range of wage variation of 16 to 24 log points associated with the differences between firms at the 90th and 10th percentiles of log value added per worker.

Theory of imperfectly competitive labor markets

Model to interpret the empirical facts:

- assume that wages set by employers to maximize profits, subject to constraints on the relationship between wages and the supply of labor.
- heterogeneity across workers in their valuation of jobs at different employers: endows firms with some power to set wages as in classic monopsony models
- can generate empirically plausible connections between firm productivity and wages
- observationally equivalent to simple rent-sharing models such as equation (1)
- under reasonable assumptions, generate the prediction that wages are additively separable in worker and firm heterogeneity

Theory of imperfectly competitive labor markets

There are J firms

- Two types of workers: Low (L) and high (H) skilled.
- Each firm j ∈ {1, ..., J} posts (w_{Lj}, w_{Hj}) skill-specific wage pair that all workers observe with no cost.
- For worker i in skill group S ∈ {L, H} the indirect utility of working at firm j is:

$$u_{iSj} = \beta_S \log(w_{Sj} - b_s) + a_{Sj} + \epsilon_{iSj}$$

where

- − b_s is skill group specific reference wage (outside option).
- $-a_{Si}$ is firm-specific amenity common to all workers in group S.
- ϵ_{iSj} is idiosyncratic preferences (distance to workplace, interactions with co-workers, etc.) drawn from type I extreme value distribution.

Theory: labor supply

Probability that random worker i with skill S chooses to work in firm j:

$$p_{Sj} = Pr(\arg\max_{k} u_{iSk} = j) = \frac{\exp(\beta_S \log(w_{Sj} - b_S) + a_{Sj})}{\sum_{k=1}^{J} \exp(\beta_S \log(w_{Sk} - b_S) + a_{Sk})}$$
$$\approx_{J \to \infty} \lambda_S \exp(\beta_S \log(w_{Sj} - b_S) + a_{Sj})$$

- **1.** Equality is famous result by McFadden (Nobel Prize in 2000) and due to extreme value assumption on ϵ_{iSi} (not proved here).
- Approximation holds when small individual firm's impact on the numerator will be negligible.

Theory: labor supply

• Let \mathcal{L} and \mathcal{H} be the total number of lower and higher skilled workers respectively.

$$\log(L_j(w_{Lj})) = \log(\mathcal{L}p_{Lj}) = \log(\mathcal{L}\lambda_L) + \beta_L \log(w_{Lj} - b_L) + a_{Lj}$$

$$\log(H_j(w_{Hj})) = \log(\mathcal{H}p_{Hj}) = \log(\mathcal{H}\lambda_H) + \beta_H \log(w_{Hj} - b_H) + a_{Hj}$$

Theory: Firm optimization

 Production Function is twice differentiable and constant returns to scale (CRS).

$$Y_j = T_j f(L_j, H_j)$$

 The firm's problem is to post a pair of skill-specific wages that minimize the cost of labor:

$$\min_{w_{Lj},w_{Hj}} w_{Lj} L_j(w_{Lj}) + w_{Hj} H_j(w_{Hj})$$

s.t.
$$T_j f\left(L_j(w_{Lj}), H_j(w_{Hj})\right) \geq Y$$

or equivalently:

$$\min_{w_{Lj},w_{Hj}} w_{Lj} \mathcal{L} \lambda_L e^{eta_L \log \left(w_{Lj}-b_L
ight) + a_{Lj}} + w_{Hj} \mathcal{H} \lambda_H e^{eta_H \log \left(w_{Hj}-b_H
ight) + a_{Hj}}$$

$$\text{s.t. } T_j f\left(\mathcal{L} \lambda_L e^{\beta_L \log\left(w_{Lj} - b_L\right) + a_{Lj}}, \mathcal{H} \lambda_H e^{\beta_H \log\left(w_{Hj} - b_H\right) + a_{Hj}}\right) \geq Y$$

Theory: Firm optimization

The associated first order conditions can be written as:

$$w_{Lj} + \frac{w_{Lj} - b_L}{\beta_L} = T_j f_1 \mu_j$$

$$\Rightarrow w_{Lj} \left(1 + \frac{w_{Lj} - b_L}{\beta_L w_{Lj}} \right) = T_j f_1 \mu_j$$

$$\Rightarrow w_{Lj} \frac{1 + e_{Lj}}{e_{Lj}} = T_j f_1 \mu_j$$

where μ_j represents the marginal cost of production, which the firm will equate to marginal revenue at an optimal choice for Y. Also $e_{Sj} = \frac{\beta_S w_{Sj}}{w_{Sj} - b_S}$ is the elasticity of labor supply w.r.t. wage.

For both skill groups, labor supply to the firm becomes infinitely elastic as wages approach the reference wage level b_S.

Theory: Firm optimization

The firm's first order conditions can be re-written:

$$\mathbf{w}_{Lj} = \frac{1}{1 + \beta_L} \mathbf{b}_L + \frac{\beta_L}{1 + \beta_L} T_j \mathbf{f}_1 \mu_j \tag{3}$$

 w_{Lj} is a weighted average of the reference wage b_L and group L's marginal revenue (= marginal factor cost; i.e. assume perfect competition in output market).

- Similarly

$$w_{Hj} = \frac{1}{1 + \beta_H} b_H + \frac{\beta_H}{1 + \beta_H} T_j f_2 \mu_j \tag{4}$$

- − As β_S → 0 firm pays the reference wage b_S .
- In the case where the reference wage is 0, the labor supply function for group S has a constant elasticity β_S and the first order condition sets the wage equal to a constant fraction $\beta_S/(1+\beta_S)$ of the marginal revenue product.

Theory: Baseline case

Linear production function and fixed output price.

$$Y_j = T_j N_j = T_j ((1 - \theta)L_j + \theta H_j)$$

- − N_i is efficiency units and $\theta \in (0.5, 1)$
- Output price is fixed and equal to P_i^0 .
- Recall μ_j is marginal cost of production and firm will equate that to the marginal revenue when solving for Y.
- Equation (3) $\Rightarrow w_{Lj} = \frac{1}{1+\beta_L}b_L + \frac{\beta_L}{1+\beta_L}T_j(1-\theta)P_j^0$
- Equation (4) $\Rightarrow w_{Hj} = \frac{1}{1+\beta_H}b_H + \frac{\beta_H}{1+\beta_H}T_j\theta P_j^0$

Theory: Baseline case

 The determination of the optimal wage in the simplified situation where there is only one skill group is illustrated in this Figure:

Determination of Wages

> marginal Wage, factor cost MRP. MRP varies MFC with VA/worker w^o inverse supply b L^{0} **Employment**

Theory: Baseline case

• If we assume reference wages are proportional to skill groups' relative productivity (i.e. $b_L = (1 - \theta)b$ and $b_H = \theta b$) we get:

$$w_{Lj} = \frac{(1-\theta)b}{1+\beta_L} + \frac{\beta_L}{1+\beta_L} T_j (1-\theta) P_j^0$$

$$\Rightarrow \log(w_{Lj}) = \log\left(\frac{(1-\theta)b}{1+\beta_L}\right) + \log(1+\beta_L R_j)$$

where $R_j = \frac{T_j P_j^0}{b}$ is the proportional gap in marginal labor productivity at firm j relative to the competitive sector.

Rent Sharing

• $v_j = \frac{P_j^0 Y_j}{N_i}$ is the value added per efficiency unit of labor.

$$v_j = \frac{P_j^0 Y_j}{N_j} = P_j^0 T_j \Rightarrow R_j = \frac{v_j}{b}$$

 The elasticity of wages of skill group S with respect to value added per worker is:

$$\xi_{Sj} = \frac{\partial \log w_{Sj}}{\partial \log v_j} = \frac{\partial \log w_{Sj}}{\partial \log R_j}$$
$$= \frac{\beta_S R_j}{1 + \beta_S R_j}$$

 Same the as the sharing elasticity w.r.t. the quasi-rent (2), where workers are assumed to capture a fixed share of the quasi-rents!

Rent Sharing

Now compare to the empirical estimates:

- The estimated rent sharing elasticities above indicate that a typical value of this elasticity is around 0.10, which suggests that the average value of $\beta_S R_i$ is also around 0.10.
 - An average worker earns about 10% higher wages than he or she would earn at the lowest-wage firms in the economy that have $v_i = b$.
 - Remarkably consistent with the estimates of Card, Cardoso and Kline (2016) obtained by benchmarking wage premiums at firms in the Portuguese economy relative to the premiums paid by the least profitable firms in the country.

Markdowns

We can rewrite elasticity of labor supply and wages as:

$$\Rightarrow e_{Sj} = \frac{\partial \log(S_j)}{\partial \log(w_{Sj})} = \frac{\beta_S w_{Sj}}{w_{Sj} - b_S} = \frac{1 + \beta_S R_j}{R_j - 1}$$
$$w_{Lj} = \frac{e_{Sj}}{1 + e_{Sj}} T_j f_1 \mu_j$$

- Suppose 20% markdown, then $e_{Sj}\approx 4$ and with $\beta_S R_j\approx 0.1$ we have: $R_j\approx 1.3$ and $\beta_S\approx 0.08$.
- Reasonable "near competitive" benchmark although, admittedly markdowns often larger (remember Yeh et al.) and many direct estimates of the elasticity of supply to the firm are lower than 4 (Manning, 2011).

Theory: relative wages

• When $\beta_L = \beta_H = \beta$ and approximating $\log(1 + \beta_L R_i) \approx \beta_L R_i$:

$$\log w_{Sj} = \log \frac{b}{1+\beta} + \mathcal{I}(S = L) \log(1-\theta) + \mathcal{I}(S = H) \log \theta + \beta R_j$$

$$\Rightarrow \log w_{Sj} = \alpha_S + \psi_j$$

where α_S is a skill-group-specific constant and $\psi_j = \beta R_j = \frac{\beta}{b} v_j$ is the firm-specific wage premium paid by firm j.

- This simple model therefore yields the additively separable formulation proposed by AKM.
- The firm effects should be strongly related to value added per worker, something we saw evidence for above.

Theory: Between-Firm Sorting

• Let π_{Si} be the share of workers in group S working at firm j.

$$\Rightarrow E(\log w_{Si}) = \alpha_S + \sum_j \psi_j \pi_{Sj}$$

$$\Rightarrow E(\log w_{Hi}) - E(\log w_{Li}) = \alpha_H - \alpha_L + \sum_j \psi_j(\pi_{Hj} - \pi_{Lj})$$

 The last term in this expression represents a between-firm sorting component of the average wage gap.

Between-Firm Sorting

• When the supply parameter β varies across groups the wage decomposition will contain an additional term, reflecting a weighted average across firms of the rent sharing components of the two skill groups:

$$E(\log w_{Hi}) - E(\log w_{Li}) = \alpha_H - \alpha_L + \sum_j \psi_j^L(\pi_{Hj} - \pi_{Lj}) + \sum_j (\psi_j^H - \psi_j^L)\pi_{Hj}$$
$$= \alpha_H - \alpha_L + \sum_j \psi_j^H(\pi_{Hj} - \pi_{Lj}) + \sum_j (\psi_j^H - \psi_j^L)\pi_{Lj}$$

As in classic Oaxaca-style decompositions, this gives an alternative way to evaluate the contributions of differences in the distributions of the two skill groups across firms, and differences in the return to working at a given firm between the two groups.

Actual sorting and rent-sharing by skill

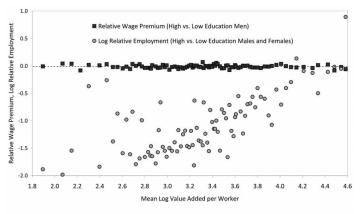


Fig. 8.—Relative wage premium and relative employment of high- versus low-education workers. Firms are divided into 100 cells on the basis of mean log value added per worker in 2005–9, with equal numbers of person-year observations per cell. A color version of this figure is available online.

Theory: Generalizations

- Downward-sloping product demand: assume firm faces an inverse demand function $P_j = P_j^0 Y_j^{\frac{-1}{\epsilon}}$ with $\epsilon > 1$. Implies monopolistic behavior but key results are the same.
- Assume that the firm's output is a CES aggregate of high and low skilled labor:

$$Y_j = T_j N_j = T_j f(L_j, H_j) = T_j \left[(1 - \theta) L_j^{\rho} + \theta H_j^{\rho} \right]^{\frac{1}{\rho}}$$

where $\rho \leq 1$ and $\sigma = \frac{1}{1-\rho}$ is the elasticity of substitution.

For details, see the paper...

Takeaway

- While the ability of firms to set wages is disciplined by market competition, there are clearly limits to those competitive forces, which also fail to eliminate productivity and output price differences across firms.
 - Firms will be able to set wages if workers differ in their valuation of firms' non-wage characteristics.
 - [Search and matching theory would provide an alternative explanation for firms' wage setting power]

Takeaway

- The idea that even highly advanced labor markets like that of the United States might be better characterized as imperfectly competitive opens a host of questions about the welfare implications of industrial policies and labor market institutions such as the trade unions, minimum wage, unemployment insurance, and employment protection.
- It affects our interpretation of the sources of inequality and its changes over time [soon...]. Also refer back to our discussion of the effect of the German minimum wage in Dustmann et al. 2021.

Basic Readings

 Card, D., Cardoso, A. R., Heining, J., & Kline, P. (2018). Firms and labor market inequality: Evidence and some theory. Journal of Labor Economics, 36(S1), S13-S70.