Labour Economics Problem Set 2: Measurement Error and IV Estimation

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Task a) Descriptive Statistics

Table: Summary Statistics (without ME)

Variable	*n*	Min	Max	Median	IQR	Mean	SD	SE	CI
education	5000	10.000	24.000	12.000	4.000	12.766	2.424	0.034	0.067
hours	5000	6.354	9.704	7.950	0.666	7.953	0.489	0.007	0.014
ln_wage	5000	2.374	4.124	3.352	0.317	3.360	0.230	0.003	0.006

Table: Summary Statistics (with ME)

Variable	*n*	Min	Max	Median	IQR	Mean	SD	SE	CI
education	5000	10.000	24.000	12.000	4.000	12.766	2.424	0.034	0.067
hours_ME	5000	6.074	9.828	7.937	0.750	7.948	0.549	0.008	0.015
ln_wage_ME	5000	0.002	6.344	3.349	1.166	3.356	0.855	0.012	0.024

Task a) Descriptive Statistics

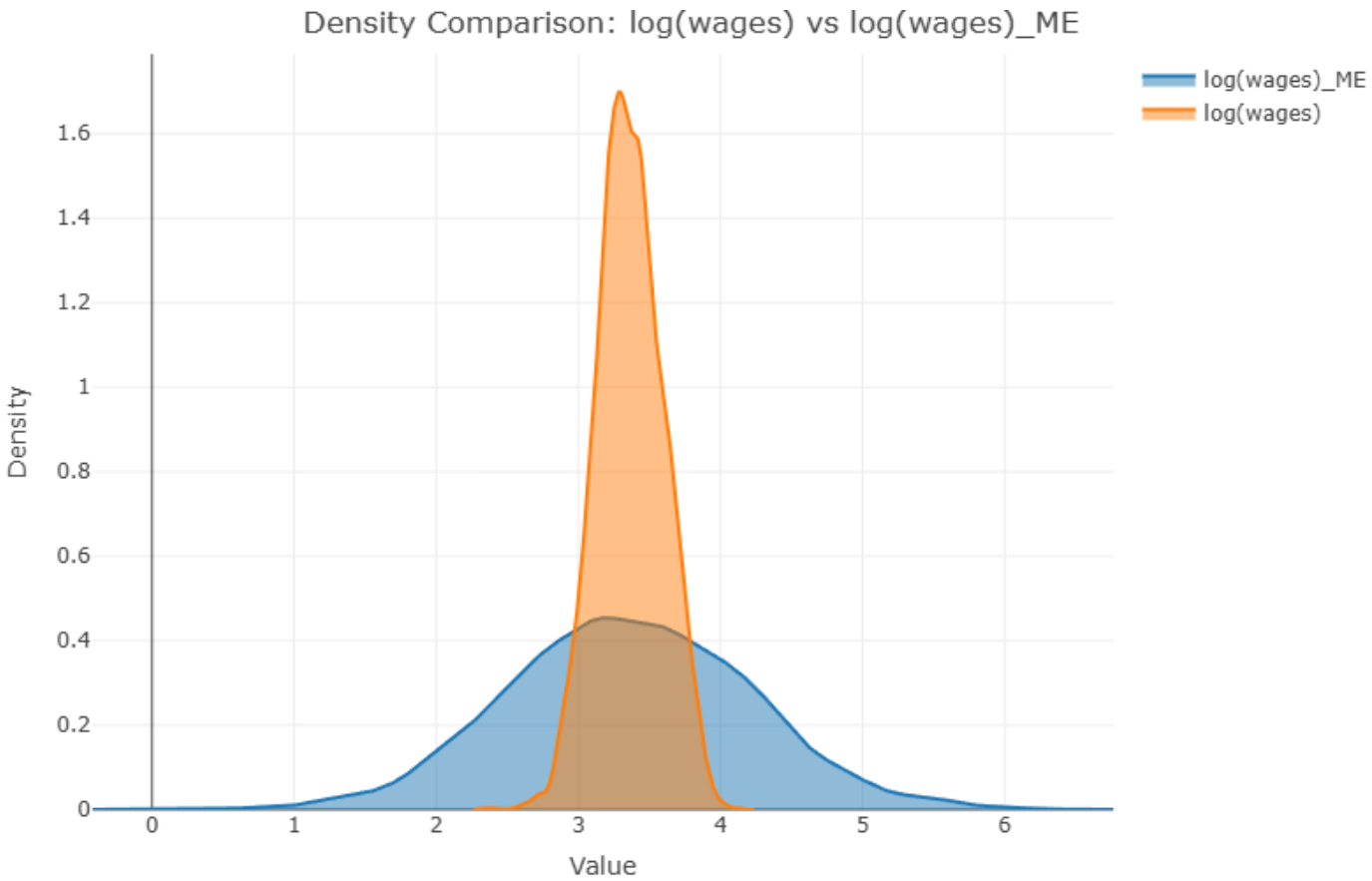

Figure: Comparison of both log(wages) densities, with and without measurement error.

Task a) Descriptive Statistics

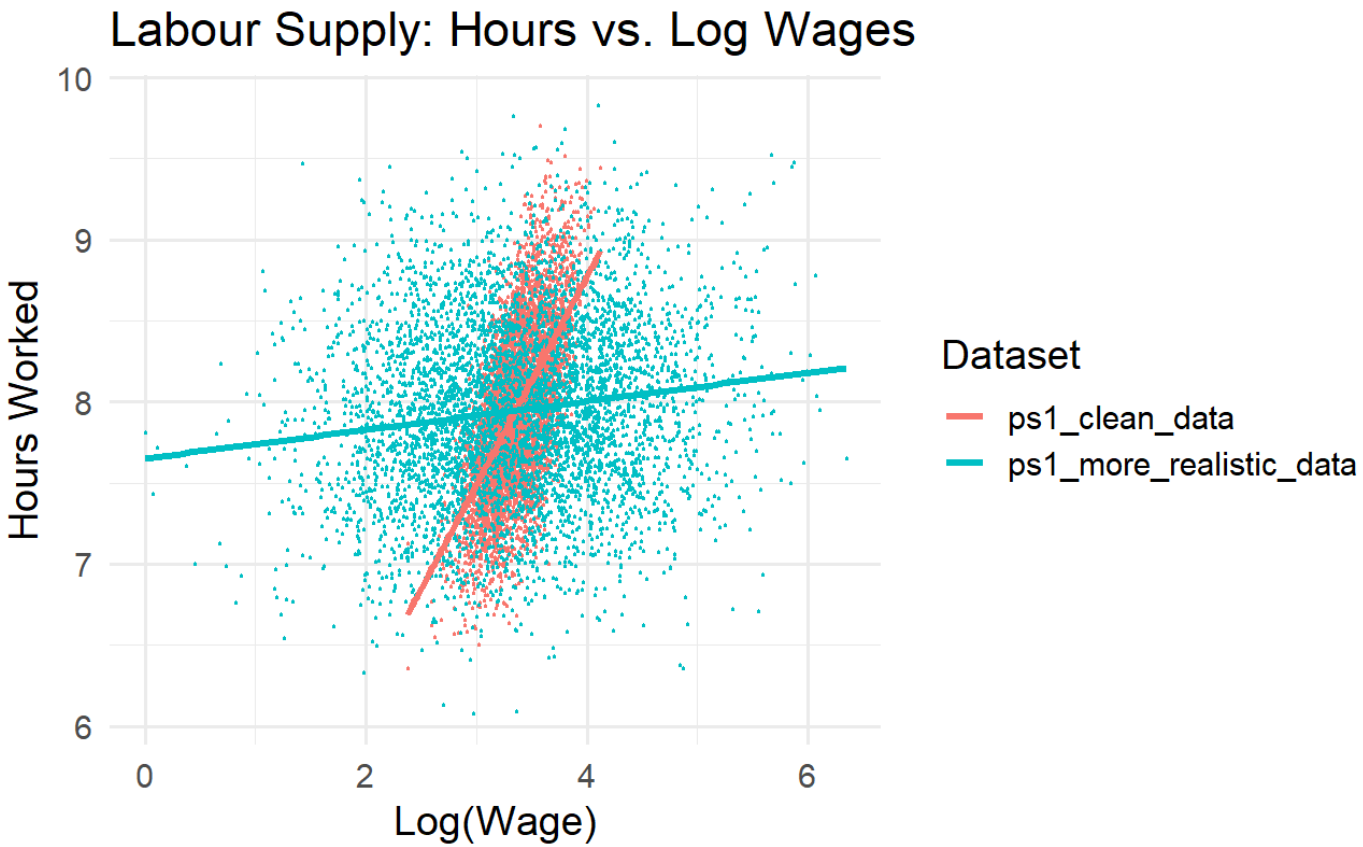

Figure: Comparison of both scatter plots. Increase in dispersion (due to measurement errors) clearly visible.

Task a) Descriptive Statistics

Key findings:

- Central tendencies are stable across datasets.
- Dispersion increases markedly in the realistic (ME) dataset.
- ME widens the range and increases variance, especially for log wages.
- Correlation between wages and hours weakens.
- Motivation is unobserved in PS2, adding potential OVB.

Task b) OLS Regression With Measurement Error

$$\boxed{\text{hours}_i^{ME} = \beta_0 + \beta_1 \ln\left(\text{wage}_i^{ME}\right) + \varepsilon_i}$$

Effects:

- Coefficient $\hat{\beta}_1$ falls from 1.285 (clean) to 0.088 (ME).
- ME in the regressor biases slopes toward zero (attenuation).
- ME in the outcome inflates residual variance (0.390 to 0.544) but does not bias $\hat{\beta}_1$.
- Explained variation R^2 drops from 0.365 to 0.019.

Task c) Balancedness Check Before Using the Instrument

Goal: Verify that the wage premium was randomly assigned. If assignment is random, pre-treatment characteristics (e.g., education) should be **balanced** across groups.

Statistical test: Two-sample t-test of mean education in groups with and without the wage premium:

$$H_0: \; E[\text{education} \mid \text{premium} = 0] = E[\text{education} \mid \text{premium} = 1].$$

Result:
t = -0.614, p = 0.539 $\Rightarrow$ **No significant difference. Education is balanced $\rightarrow$ assignment behaves like random.**

Task d) Balance Check on Motivation (PS1)

Now we are allowed to use the unobserved characteristic motivation for a second balancedness check. Same t-test approach yields

Result:
t = -0.97, p = 0.33 $\Rightarrow$ **No significant difference. Motivation is balanced $\rightarrow$ taken together with part c), this is consistent with random assignment and the earlier balance check on education.**

Task e) IV Using Wage Premium

First-stage regression:

$$\boxed{\ln\left(wage_i^{ME}\right) = \pi_0 + \pi_1 \cdot premium_i + e_i}$$

Results:

- Strong instrument: $F = 200.6$.
- Premium increases log wage by 0.345 ($\approx$42%).

Second stage:

$$\boxed{hours_i^{ME} = \beta_0 + \beta_1 \ln(\widehat{wage_i}) + u_i}$$

- IV coefficient $\hat{\beta}_1 = 0.235$ ($p < 0.001$). Elasticity interpretation: 1% increase in wages raises hours by about 0.235%.
- Coefficient is close to the full model specification obtained in sheet 1 ($\hat{\beta}_1 = 0.210$).

Task f) Differences to Question d) in Sheet 1

- In the original regression of hours on ln(*wage*), the coefficient suffered from **omitted variable bias**.
- Motivation and education were omitted and their effects were falsely attributed to ln(*wage*), inflating the OLS estimate.
- Using the random assigned wage premium as an instrument isolates **exogenous variation** in wages.
- The instrument is uncorrelated with motivation and education (Balancedness check).
- If relevance, exogeneity and exclusion hold: IV estimator removes OVB (and ME!) and delivers an actual estimate of the causal effect, without including characteristics into the regression.

Task g) Extended First Stage With Education

Extended first-stage specification:

$$\boxed{\ln\left(wage_i^{ME}\right) = \pi_0 + \pi_1\, premium_i + \pi_2\, education_i + e_i}$$

Implications:

- IV estimate remains stable: $\hat{\beta}_1 = 0.212$
- Education explains unrelated part of the variation in wages and hours, thereby improving precision.
- Standard error of $\hat{\beta}_1$ shrinks (0.047 to 0.040).
- Causal conclusion is not affected, because including education only absorbs variation in wages and hours that is unrelated to the instrument (Exogeneity of $premium_i$).

Task h) Preferred Estimate of Labour Supply Elasticity

Comparison of OLS vs IV:

- Full OLS (no measurement error, includes education and motivation): $\hat{\beta}_{\ln wage} \approx 0.21$
- IV using wage premium (with measurement error and OVB): $\hat{\beta}_{\ln wage} \approx 0.235$
- Both estimates are very similar $\rightarrow$ consistent elasticity estimate.

Interpretation:

- Using IV confirms the causal effect of wages on labour supply.
- Elasticity: a 1% increase in wages raises hours worked by $\approx$ 0.21–0.24%.
- IV is preferred in real-world settings: accounts for measurement error and omitted variables, which are very likely to occur.