technische universität
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Lecture 3a:
Comparative Statics of the Demand of Labour

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Comparative Statics of the Demand of Labour

Overview

- The broadest definition of the demand for labour involves any decision made by an employer regarding its workers, their employment, compensation, training.
- In Marshall's statement of neoclassical economics (1920), much of the focus in analysing labour markets was the employer's decisions about how many workers to employ and how many hours the employees should work.
- Responses of this to external shocks have been the focus of the analysis. That is, the comparative statics of employers' responses to changes in product demand and factor prices.

Overview

By comparison to labour supply, issues related to labour demand occupied a less voluminous part of the field of labour (Robert Willis 1986). This has changed in recent decades, due to:

- An increased theoretical interest for the firm's internal labour market or personnel economics.
- The availability of employer-employee linked data-bases, as well as employer based surveys.

Overview

- Increased government interventions that change the incentives facing employers making decisions about employment and hours, such as
  - minimum wages
  - overtime pay
  - subsidized training
  - family leaves
  - hiring subsidies
- An increased interest in technological change, especially skill-biased technological change and recent data-driven technological changes, as well as outsourcing and offshoring.

Overview

- The ultimate goal is to show how the parameters describing the employers' long-run demand for labour can be inferred from data characterizing their employment, wages, product demand, and in some cases, the prices and quantities of other inputs.
- Our study of the static theory of the firm's labour demand will mostly focus on issues of substitution among inputs into production: capital vs. labour, low skilled vs. high skilled labour.
  - These substitution effects are at the heart of the theory of the skill premia, that has been paramount in explaining the changes in wage inequality.

Overview

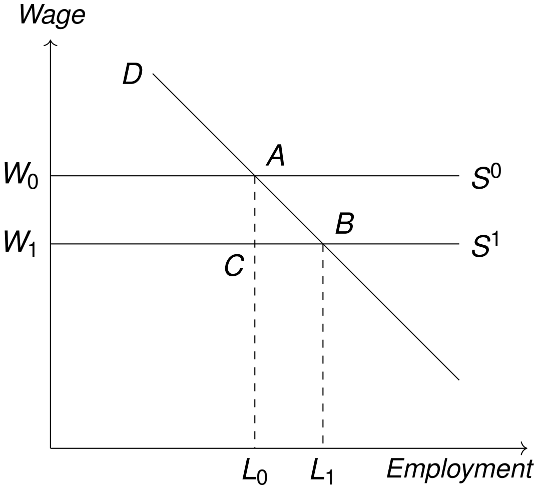
First, consider polar views of the labour-demand curve, i.e., whether the firm faces an inelastic or elastic labour supply:

1. the wage is exogenous and thus appears fixed to the individual firm, which faces an infinitely or perfectly elastic supply (horizontal supply curve).

- A wage shock will result in an adjustment of employment (e.g. a higher minimum wage will result in less employment).
- In this case (perfect competition), wage elasticities of labour demand will allow one to infer the effects of exogenous changes in wage rates.

Overview

1. Infinitely Elastic Supply



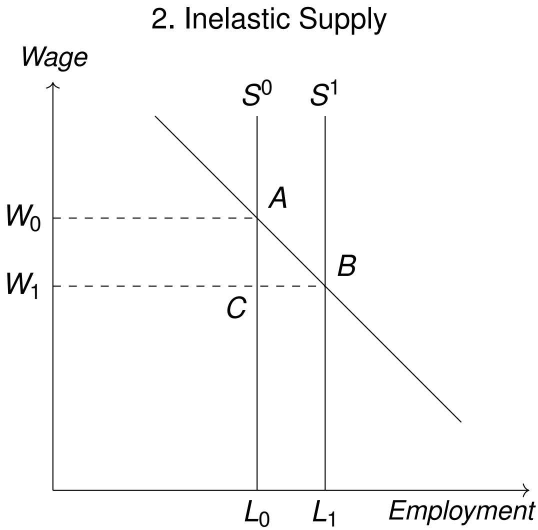
Overview

2. the supply of a particular type of labour is exogenous and thus appears fixed to the firm, which faces an inelastic supply curve (vertical supply curve).

- A supply shock will result in an adjustment of employment (e.g. the black plague lead to an increase in wages).
- Again this case (e.g. long run, full-employment) knowing the slope of the demand curve provides the information needed to infer the effect of the shock.

Note: differently from the labour supply slides, variable L now denotes labour hired / employment at the firm.

Overview

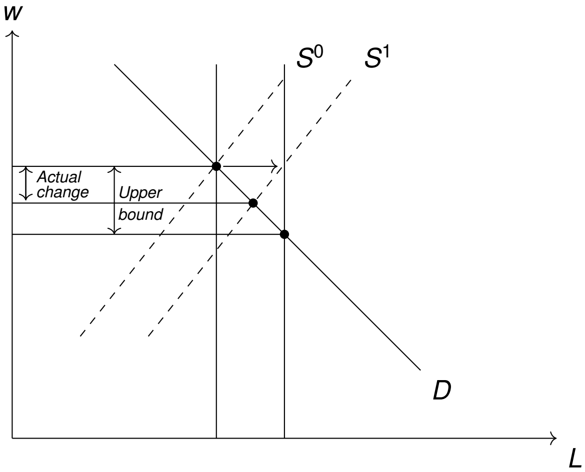


Overview

- In general, neither perfectly elastic nor completely inelastic supply characterizes labour markets. Instead, the supply of labour has a positive slope.
- So without knowing the slopes of both the demand and the supply curves, one cannot infer the actual size of changes in wages and employment to demand or supply shocks.
- However, from the slope of the demand curve, one can find some upper bounds to the impact of shocks. (see diagram)
  - When supply is assumed exogenous, bounds to changes in wages can be inferred.
  - When wage is assumed exogenous, bounds to changes in employment can be inferred.

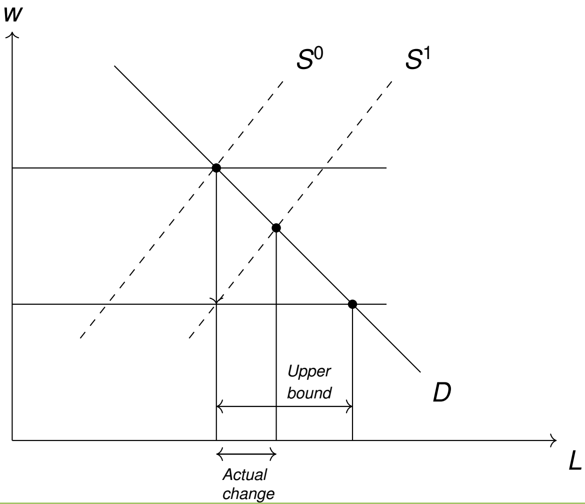
Overview

1. Supply shock, S exogenous



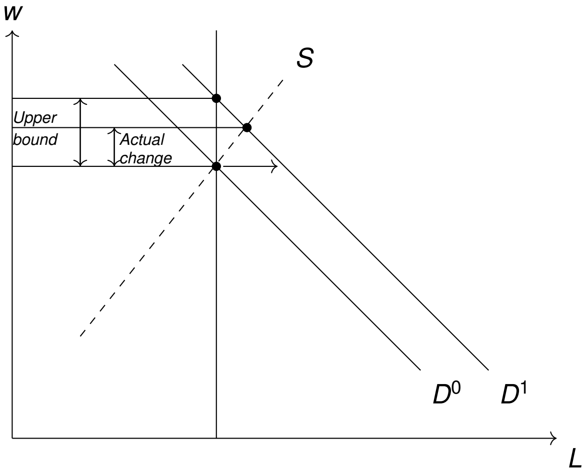
Overview

2. Supply shock, w exogenous



Overview

3. Demand shock, S exogenous



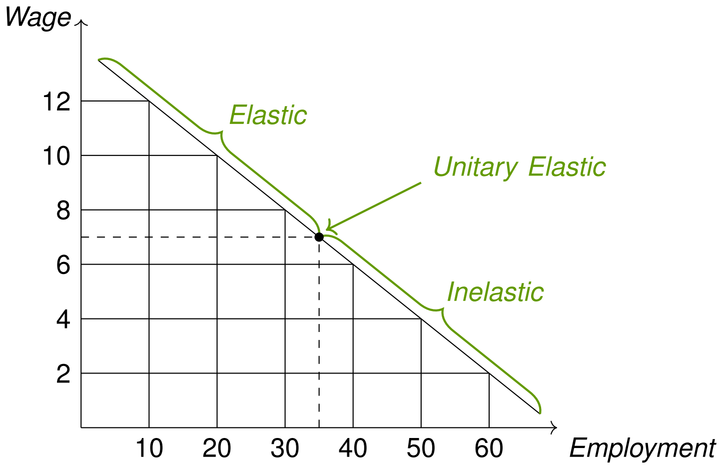
Overview

Own-wage elasticity: percentage change in labour demanded per percentage change in wage

$$\eta_{ii} = \frac{\%\Delta L_i}{\%\Delta W_i}$$

- The own-wage elasticity is negative, so we write
  - $|\eta_{ii}| > 1$ Elastic $\%\Delta L_i > \%\Delta W_i$ (greater than proportional)
  - $|\eta_{ii}| = 1$ Unit Elastic $\%\Delta L_i = \%\Delta W_i$ (proportional)
  - $|\eta_{ii}| < 1$ Inelastic $\%\Delta L_i < \%\Delta W_i$ (less than proportional)

Overview


Wage
12
10
8
6
4
2
10 20 30 40 50 60
Employment
Elastic
Unitary Elastic
Inelastic

Downward Sloping Demand Curve

Though the classic theorems of labour demand require that there are at least two inputs in production, the motivation for the downward sloping labour demand curve can be derived when only one input is assumed (Hamermesh, 1996).

- Assume that output Y is obtained from a production process described by a function that transforms labour services (L) into output:

  $$Y = F(L) \quad \text{where } F_L > 0, F_{LL} < 0$$

  that is, there are diminishing returns to the single input (in the background, all other inputs are fixed in the *short-run*).

Downward Sloping Demand Curve

- Assume that the firm is competitive in all markets and attempts to maximize profits

$$\pi(L) = pF(L) - W \cdot L \quad \text{where } F_L > 0, F_{LL} < 0$$

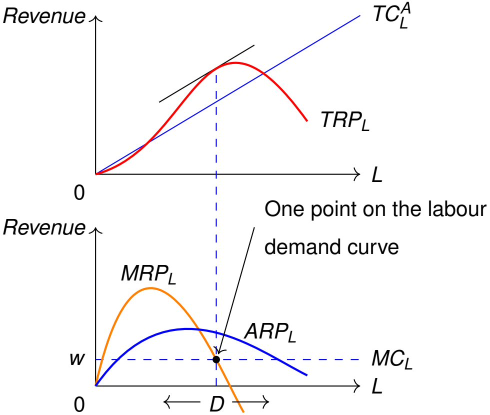
- So that the FOC condition is $F_L(L) - w = 0$, where $w = W/p$ is the real wage and the SOC requires $F_{LL} < 0$.
- Differentiation of the FOC with respect to w,

$$F_{LL}(L^*)\frac{dL^*}{dw} - 1 = 0 \Rightarrow \frac{dL^*}{dw} = \frac{1}{F_{LL}(L^*)} < 0$$

Downward Sloping Demand Curve

- Thus the more rapidly diminishing are the returns to labour, the steeper the demand curve for labour.
- Short-run labour demand is a decreasing function of labour cost
  - Here essentially a scale effect as falling p would lead to the same conclusion.
  - In the figure, rising $w = \frac{W}{p}$ would lead to an upward shift of the MC and steepening of the TC curves.
- In the longer run, the firm may contemplate replacing parts of its workforce with machines, depending on technical feasibility. This leads to an even stronger decline of labour hired (long-run elasticity larger).

Downward Sloping Demand Curve



"Labour will be hired as long as the revenue that an extra unit of labour generates is greater than the cost of that extra unit of labour."

Elasticity of Substitution

Many interesting insights from neoclassical production theory come from examining the demand for homogeneous labour in the case of two inputs.

- Assume that production exhibits constant returns to scale (CRS), as described by a linearly homogeneous function, $\delta Y = F(\delta L, \delta K)$, such that

$$Y = F(L, K) \quad \text{where } F_i > 0, F_{ii} < 0, F_{ij} > 0 \qquad i = K, L$$

  where Y is output, L is labour and K is capital.
- Long-run setting (L & K vary), firm remains competitive in the input market.

Elasticity of Substitution

- Assuming that the firm maximizes profits $\pi = Y - wL - rK$, where w is the exogenous wage, r is the exogenous price of capital services, and the price of output has been normalized to one, subject to the technology.
- The FOCs will be: $F_L = w$ and $F_K = r$, yielding the familiar statement that the ratio of the values of the marginal product, VMP, the marginal rate of technical substitution equals the factor price ratio for a profit maximizing firm.

$$MRTS_{KL} = \frac{VMP_L}{VMP_K} = \frac{1 \cdot F_L}{1 \cdot F_K} = \frac{w}{r}$$

Elasticity of Substitution

- In the two factor linearly homogeneous case, the elasticity of substitution is:

$$\sigma = \frac{d(K/L)/(K/L)}{d(F_L/F_K)/(F_L/F_K)}\bigg|_{Yconstant} = \frac{d\ln(K/L)}{d\ln(F_L/F_K)}\bigg|_{Yconstant}$$

- In the case above (firm competitive in the input markets):

$$\sigma = \frac{d(K/L)(K/L)}{d(w/r)(w/r)}\bigg|_{Yconstant} = \frac{d\ln(K/L)}{d\ln(w/r)}\bigg|_{Yconstant}$$

- Intuitively, this elasticity measures the ease of substituting one input for the other when the firm can only respond to a change in one or both of the input prices by changing the relative use of two factors without changing output.

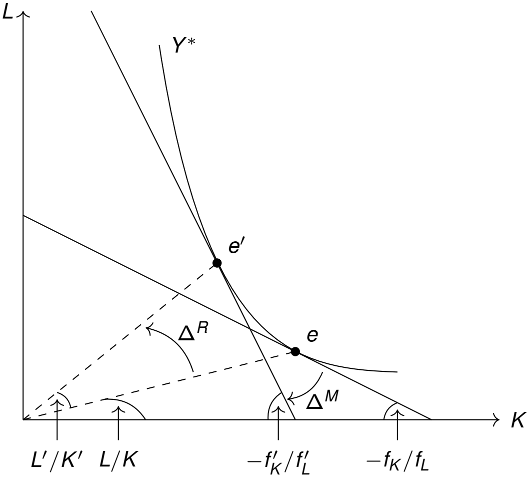
Elasticity of Substitution

Example with the Cobb-Douglas technology, $Y = AL^{\alpha}K^{(1-\alpha)}$:

- the profit maximizing condition becomes: $\frac{F_L}{F_K} = \frac{\alpha}{(1-\alpha)}\frac{K}{L} = \frac{w}{r}$
- Taking the logarithms gives $\ln\left(\frac{K}{L}\right) = \alpha' + \ln\left(\frac{w}{r}\right)$, where α' is a constant.
- Taking the differential on both sides

$$d\ln\left(\frac{K}{L}\right) = d\ln\left(\frac{w}{r}\right) \Rightarrow \frac{d\ln(K/L)}{d\ln(w/r)} = 1 = \sigma$$

Elasticity of Substitution



Elasticity of Substitution

From the figure, we see that:

- The elasticity of substitution compares the movement in the chord from L/K to L'/K' (denoted by Δ^R in the Figure) to the movement in the MRTS from F_K/F_L to F_K'/F_L' (represented by Δ^M).
- The elasticity of substitution is thus, intuitively speaking, merely $\sigma = \Delta^R/\Delta^M$.

Elasticity of Substitution

- In the case of a linear homogeneous production function (CRS) it can be shown that in general

$$\sigma = \left.\frac{F_L F_K}{Y F_{LK}}\right|_{Yconstant} \quad (1)$$

- Equation (1) shows that σ is always non-negative. The value of F_{LK} depends on the shape of the production function, but is always positive under usual production function assumptions.
- The elasticity of substitution becomes a property of the curvature of the isoquant and is thus always positive. The larger the value of σ, the flatter the constant product curve (isoquant) and the more slowly does the marginal rate of substitution increase as K is substituted for L.

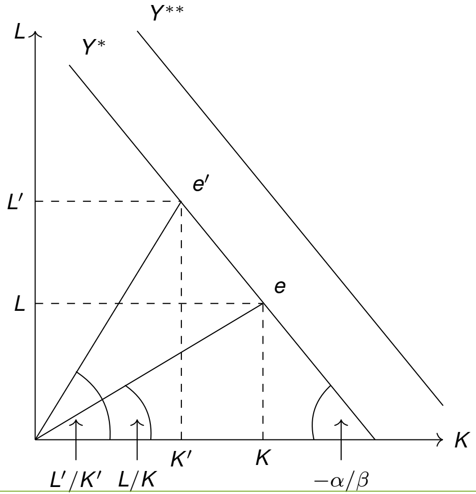
Elasticity of Substitution

There are two limiting cases:

1. If K and L are perfect substitutes, so that constant product is maintained by increasing K in proportion as L is decreased, then the isoquant is a straight line and $d^2K/dL^2 = 0$ and σ is infinite.
2. If K and L are entirely incapable of substitution, being needed in a fixed proportion, then an increase in one of the factors from this proportion must leave the product unchanged. The isoquant has a right angle and $\sigma = 0$.

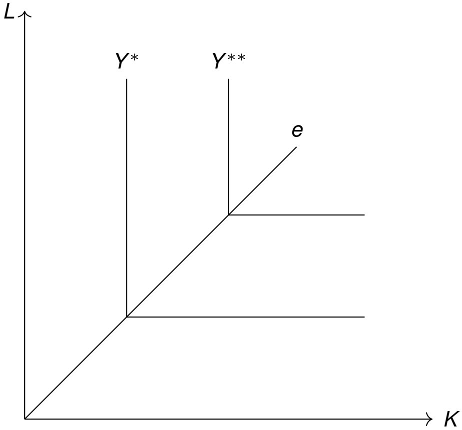
Elasticity of Substitution

1. Case: K and L are perfect substitutes



Elasticity of Substitution

2. Case: K and L are entirely incapable of substitution



Cross-Elasticities and Own-Price Elasticities

Conditional demand, i.e. holding output constant:

- The cross-elasticities of labour demand with respect to the price of capital services or of the demand for capital services with respect the wage rate are found from the comparative statics of cost-minimization:

$$\bar{\eta}_r^L = \frac{r}{L}\frac{\partial L}{\partial r} \text{ and } \bar{\eta}_w^K = \frac{w}{K}\frac{\partial K}{\partial w}$$

- A firm chooses L and K to minimize costs subject to a particular value of output:

$$\min_{L,K} C = wL + rK \text{ subject to } Y = F(K, L)$$

Cross-Elasticities and Own-Price Elasticities

- After solving for L^*, K^* from the first order conditions,

$$\frac{F_L(L^*, K^*)}{F_K(L^*, K^*)} = \frac{w}{r} \text{ and } F(L^*, K^*) = Y$$

- we can get a function that minimizes costs given a certain level of production, subject to w, r, and Y:

$$C^* = C(w, r, Y) = w \cdot L(w, r, Y) + r \cdot K(w, r, Y)$$

where $L(w, r, Y)$ and $K(w, r, Y)$ are the conditional (on Y) demand for labour and for capital, respectively.

Cross-Elasticities and Own-Price Elasticities

- This is the cost function, which has several useful properties that are derived from the assumptions about the production function and the firm's optimizing behaviour.
- It is increasing in the factor prices, $C_w > 0$, $C_r > 0$, concave, $C_{ww} < 0$, $C_{rr} < 0$ and homogeneous of degree 1 in (w, r).
- Optimal levels for labour and capital demanded are equal to their respective partial derivatives [Shephard's Lemma]:

$$L^* = C_w(w, r, Y) \text{ and } K^* = C_r(w, r, Y)$$

Cross-Elasticities and Own-Price Elasticities

- Differentiation gives us conditional labour demand is decreasing with the price of this factor $\frac{\partial L^*}{\partial w} = C_{ww} < 0$ shows symmetric cross-price effects

$$\frac{\partial L^*}{\partial r} = \frac{\partial K^*}{\partial w} = C_{wr} > 0$$

- It can be shown using Shephard's Lemma and the homogeneity of degree 1 of the cost function (see Cahuc and Zylberberg, 2014, appendix 7.2) that

$$\sigma = \frac{CC_{wr}}{C_w C_r} \tag{2}$$

where it represents the elasticity of the ratio $\frac{L^*}{K^*}$ in relation to the relative cost $\frac{w}{r}$.

Cross-Elasticities and Own-Price Elasticities

- We are now able to obtain expressions for the cross-elasticities above in terms of σ. First, we can write

$$\bar{\eta}_r^L = \frac{r}{L}\frac{\partial L}{\partial r} = \frac{r}{L^*}C_{wr} \text{ which leads to } \bar{\eta}_r^L = \frac{rC_w C_r}{L^* C}\sigma$$

- Using the labour share $s \equiv wL^*/C$, which implies $(1-s) = rK^*/C$, and the fact $L^* = C_w$ and $K^* = C_r$, leads to

$$\bar{\eta}_r^L = (1-s)\sigma \tag{3}$$

- The intuition for including $(1-s)$ here is that if capital's share is very small, a 1 percent change in its price cannot induce a large percentage change in labour demand.

Cross-Elasticities and Own-Price Elasticities

- There also exists a link between the own-price elasticity $\bar{\eta}_w^L$ and the elasticity of substitution σ.
- The conditional demand for labour depending only on Y and on the ratio $\frac{w}{r}$, we have $\frac{\partial L^*}{\partial w} = -\frac{r}{w}\frac{\partial L^*}{\partial r}$ and consequently

$$\bar{\eta}_w^L = -\bar{\eta}_r^L = -(1-s)\sigma \tag{4}$$

Note: $C_w(w, r, Y) = C_w(1, \frac{r}{w}, Y)$ as the derivative of a homogenous degree k function is homogenous of degree $k-1$. Then, $\frac{\partial L^*}{\partial w} = \frac{\partial C_w(1, \frac{r}{w}, Y)}{\partial \frac{r}{w}} \cdot (-\frac{r}{w^2})$ and $\frac{\partial L^*}{\partial r} = \frac{\partial C_w(1, \frac{r}{w}, Y)}{\partial \frac{r}{w}} \cdot (\frac{1}{w})$.

Cross-Elasticities and Own-Price Elasticities

- Intuitively, $\bar{\eta}_w^L$ is smaller in absolute value (less negative) for a given technology σ when labour's share is greater, because there is relatively less capital toward which to substitute when the wage rises.
- *Exercise*: show that $\bar{\eta}_w^K = s \cdot \sigma$

⇒ Note again that (3) and (4) reflect only substitution along an isoquant.

⇒ More realistic analyses should also include scale effects.

Unconditional Demand

- The scale effect, which is analogous to the income effect in labour supply, arises from unconditional demand. It will depend
  - on the (absolute value) of the elasticity of product demand,
  - and on the share of labour in total costs (which determines the percentage increase in price).
- Let Y^* be the level of output that maximizes profit

$$\pi(w, r, Y) = p(Y)Y - C(w, r, Y)$$

- Under perfect competition in output market, $p = C_Y(w, r, Y)$ s.t.

$$\pi_w = (p - C_Y)\frac{\partial Y}{\partial w} - C_w = -C_w$$

  which gives Hotelling's lemma: $\pi_w = -L^*$.

Unconditional Demand

- Therefore, also the unconditional labour demand function $L^* = C_w(w, r, Y^*)$
- Differentiating with respect to w, we get $\frac{\partial L^*}{\partial w} = C_{ww} + C_{wY} \frac{\partial Y^*}{\partial w}$
- Multiplying both sides by $\frac{w}{L^*}$ write this in terms of the elasticities η_W^L and η_W^Y,

$$\eta_w^L = \frac{w}{L^*} C_{WW} + \left(\frac{C_{WY} Y^*}{L^*} \right) \eta_W^Y$$

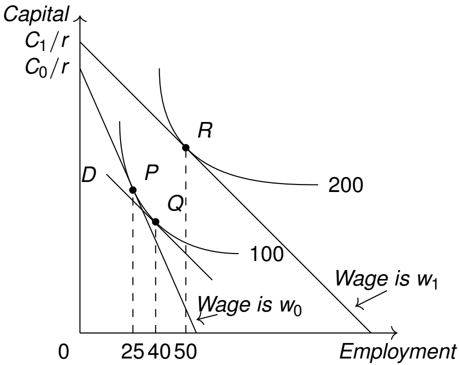
Unconditional Demand

- But the first term is just the conditional labour demand elasticity, $\bar{\eta}_w^L = \frac{w}{L^*} C_{ww} < 0$, and the second is the elasticity of labour demand with respect to output, $\bar{\eta}_Y^L = \frac{C_{wY} Y^*}{L^*}$, we get

$$\eta_w^L = \bar{\eta}_w^L + \bar{\eta}_Y^L \eta_w^Y$$

- The relation show the different effects of a rise in wage on the demand for labour.
  - First term is the substitution effect: a rise in the cost of labour always leads to a reduced utilization of this factor.
  - Second term is a scale effect, which is always negative ($\bar{\eta}_w^Y$ is always of opposite sign to C_{wY} from SOC) and accentuates the substitution effect.

Unconditional Demand



Substitution and Scale Effects

A wage cut generates substitution and scale effects. The substitution effect (the move from point P to point Q) encourages the firm to use a more labor-intensiv method of producing a given level of output. The scale effect (from Q to R) encourages the firm to expand, further increasing the firm's employment.

Unconditional Demand

- Under constant returns to scale and perfect competition, scale effects can be expressed as a function of the labour share of total cost and of the elasticity of product demand, we get (not so easily, see Dixit (1976, p.79))

$$\eta_w^L = \underbrace{-(1-s)\sigma}_{substitution} \underbrace{-s\eta_p^Y}_{scale} \tag{5}$$

- This is known as the fundamental law of factor demand: it divides labour demand elasticity into substitution effect (holding output constant) and scale effect (holding relative inputs constant).

Elasticity of product demand: $\eta_p^Y = -\frac{\partial Y}{\partial p}\frac{p}{Y} > 0$

Unconditional Demand

Interpretation of scale effect:

- When wages increase, production costs rise and this raises product prices. If the elasticity of demand for the product is large then there will be large declines in output following price increases.
  - Greater the decrease in output, greater is the decline in employment of labour.
- Output with a high share of labour will be affected more. If labour's share in total costs is only 20% then a 10% increase in the wage rate will raise costs by 2%. But if share is 80% then the same 10% increase in wages raises costs by 8%.
  - The greater the share of labour in total costs, the higher the elasticity of labour demand.

Unconditional Demand – Important Extensions

L is not a homogeneous factor in reality:

- E.g. low vs. high-skilled workers, white vs. blue-collar workers.
- Extend framework to cover multiple types of labor (e.g. important when thinking about wage inequality).

In reality, *L* does not directly fully adjust to new optimum:

- Dynamic theory: labor demand with adjustment costs.
- Adjustment costs of explicit or implicit nature, e.g.: Recruitment costs/costs of integrating workers into the production process.
- Costs dampen reaction to change in factor prices in short run (similar to what we've seen for fixed capital above).

Hicks-Marshall Rules of Derived Demand

- Alfred Marshall (1920) used four “laws” to summarize the effects of factors that influence the own-wage elasticity.
- The first three laws can be seen as working through the expressions for η_w^L and $\bar{\eta}_w^L$ in (5).

Marshallian Law of Labour Demand:

- Other things being equal, the own-wage elasticity of labour demand η_w^L will be greater (that is, will display a larger reduction in employment in response to a wage change), the higher is

Hicks-Marshall Rules of Derived Demand

1. substitutability of other factors of production: the more easily the other factors can be substituted for labour (comes from the proportionality to σ)
2. price elasticity of the relevant product demand: the more elastic is the demand for the product (increases with η_P^Y)
3. cost share of labour in total production costs: the greater the share of employing the type of labour in the total cost of production (proportionality to s; holds only when $\eta_P^Y > \sigma$).
4. supply elasticity of other factors of production: that is, usage of the other factors of production can be increased without substantially increasing their prices (relaxes the maintained assumption of a constant r).

Estimating the Elasticity of Labour Demand

- The “game” of estimating these elasticities is to propose a production function that ameliorates the estimation process. For example, forget using Cobb-Douglas: the elasticity of substitution is fixed at one.
- As example, another production function is the Constant Elasticity of Substitution function (CES), which, as you might guess from the name, the elasticity of labour demand does not depend on current production, or costs. The CES function is:

$$Y = [\alpha L^{\rho} + (1-\alpha)K^{\rho}]^{1/\rho}$$

- The marginal products are $\frac{\partial Y}{\partial L} = \alpha\left(\frac{Y}{L}\right)^{1-\rho}$, $\frac{\partial Y}{\partial K} = (1-\alpha)\left(\frac{Y}{K}\right)^{1-\rho}$

Estimating the Elasticity of Labour Demand

- So that
$$\frac{F_K}{F_L} = \frac{1-\alpha}{\alpha}\left(\frac{L}{K}\right)^{1-\rho} = \frac{r}{w}$$
taking the logarithm and differentiating with respect to $\ln(w/r)$ gives
$$\frac{\partial \ln(L/K)}{\partial \ln(w/r)} = \sigma = \frac{1}{1-\rho}$$
- We could try to estimate this equation, adding an error term $\ln(L/K) = \beta_0 + \beta_1 \ln(w/r) + \varepsilon$, where an estimate of the constant-output elasticity of labour demand would be $\hat{\beta}_1$.

Estimating the Elasticity of Labour Demand

- This specification seems restrictive but also convenient: the elasticity does not depend on the current level of production, or the current relative use of each factor.
- Note, if the price of capital is constant, we are in effect estimating a regression equation similar to one seen before for labour supply.
- We need some way of determining whether wage fluctuations are due to exogenous changes in labour supply, or exogenous changes in labour demand (simultaneity bias).
- As in labour supply, we also need to assure no unobserved heterogeneity (omitted variables bias): certain types of labour may be higher-skilled, get higher wages, and be more demanded.

Estimating the Elasticity of Labour Demand

- Cahuc & Zylberberg quoting Hamermesh (1996)
  Unconditional $\eta_w^L = -1.0$
  Conditional $\bar{\eta}_w^L = -0.3[-0.15, -0.75]$
- Given that as shown earlier,

$$\bar{\eta}_w^L = -(1 - s)\sigma$$

  where s is share of labour in total cost and σ is the elasticity of substitution of labour and capital.
- Given conditional elasticity of -0.3 and known share of labour of 0.7 in most advanced countries, then a reasonable elasticity of substitution between capital and labour is $\sigma \approx 1.0$, very close to the Cobb–Douglas with $\alpha = 0.7$.

Basic Readings

- Hamermesh, D.S. Labor Demand, Princeton University Press, 1996, chap. 2, 3.
- Hamermesh, D.S. “The Demand for Labor in the Long Run,” in Ashenfelter, O.C. and R. Layard, editors, Handbook of Labor Economics, North-Holland, vol I, 1986, chap. 8.
- Cahuc, Carcillo and Zylberberg. *Labor Economics*. Chapter 2.

Appendix: Applications of Marshall's Law

1) The elasticity of labour demand is greater, the greater the substitutability of competing production factors
   - η_w^L is higher when the elasticity of substitution is higher
   - η_w^L is higher in the long run (when choice of technology is endogenous)
   - η_w^L is higher for deregulated labour markets
2) The elasticity of labour demand is greater, the higher is the price elasticity for the relevant product whose production gives rise to the demand for labour
   - η_w^L is higher for individual firms than for industries
   - η_w^L is higher in the long run
   - η_w^L is higher for exporting firms

Appendix: Applications of Marshall's Law

3) The elasticity of labour demand is greater, the higher the cost share of labor in total production costs
   - η_w^L is higher in services than in manufacturing sectors with high labour share in value-added or with lower cost shares of material inputs

(4) The elasticity of labour demand is greater, the higher the supply elasticity of competing factors
   - η_w^L is higher in countries that are open to capital mobility
   - η_w^L is higher in countries that are open to labour migration from abroad
   - η_w^L is higher in the long run (factor mobility takes time)