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Lecture 2b:
Estimation of labour supply elasticities

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Empirical Economics

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Estimation–from previous lecture

- Suppose that we have individual data on hours of work H_i, the wage rate w_i, and on non-labor income Y_i, we could estimate a simple OLS regression

$$H_i = \beta_0 + \beta_1 w_i + \beta_2 Y_i + \varepsilon_i$$

Estimation–from previous lecture

- Then the estimated effects:
  - $\hat{\beta}_1$ will be the overall (uncompensated) effect, $\frac{\partial H}{\partial w}$
  - $\hat{\beta}_2 \overline{H}$ will be the income effect, $\overline{H} \frac{\partial H}{\partial Y}$, (evaluated at mean hours)
  - $\hat{\beta}_1 - \hat{\beta}_2 \overline{H}$ will be the substitution effect, $\frac{\partial H}{\partial w} - \overline{H} \frac{\partial H}{\partial Y}$, (evaluated at mean hours)
  - $\hat{\beta}_2 \frac{\overline{Y}}{\overline{H}}$ will be the income elasticity of labour supply, $\frac{\partial H}{\partial Y} \frac{Y}{H}$, (evaluated at mean hours and mean income)

Outline

1. Problems with the OLS Estimation of Labour Supply Functions:
   - Unobserved heterogeneity/endogeneity of wage rate
   - Measurement error in wages and division bias
   - (Selection into labor force/unobserved wage rate for non-labour market participants–not covered here)
2. Using Taxes & Transfers Programs to Estimate Labour Supply
3. Overview of Findings in the Literature.

Labour supply estimation

- Elasticity of labour supply: Responsiveness of hours of work to changes in the wage rate:

$$\varepsilon_{Hw} = \frac{\partial H}{\partial w}\frac{w}{H} \approx \frac{\Delta H}{\Delta w}\frac{w}{H} = \frac{\frac{\Delta H}{H}}{\frac{\Delta w}{w}}$$

- That is, percent change in hours worked divided by the percent change in wage rate.
  - If a worker's wage is initially 20$ per hour and she works 1,900 hours per year. After she gets a raise to 24$ per hour, she decides to work 1,938 hours per year $\rightarrow$ 2%/20%=0.1

Labour supply estimation

- When we run regressions where the outcome (e.g. hours) and the treatment (e.g. wages) are in logarithms, can the resulting estimates be interpreted as elasticities?
  - $\frac{\partial \ln(H)}{\partial \ln(w)} =$
- What other conditions/assumptions are needed?

Unobserved heterogeneity

Unobserved heterogeneity/endogeneity of wage rate:

- Recall the labour supply function

$$H_i = \beta_0 + \beta_1 w_i + \beta_2 Y_i + \beta_3 X_i + \varepsilon_i$$

- Workers with higher wages may have higher labour force attachment; put more effort, more hours.
- More educated (in X_i) and more motivated (μ_i) workers get higher wages
  - e.g. do not observe motivation s.t. $\varepsilon_i = \mu_i + \nu_i$ with ν_i iid.
  - $Corr(w_i, \varepsilon_i) \neq 0$ and β_1 will be (upward?) biased

Unobserved heterogeneity

- The bias is

$$p\lim(\hat{\beta}_1^{OLS}) = \beta_1 + \frac{Cov(\widetilde{w}_i, \varepsilon_i)}{Var(\widetilde{w}_i)}$$

- Controlling for X_i's helps, but it may not be sufficient to remove all omitted variables bias.
- $\widetilde{w}_i$ here is the residual from regressing w_i on Y_i and X_i (Regression Anatomy or Frisch-Waugh-Lovell theorem).
- Refresh your knowledge of this in Angrist and Pischke (2008) book if necessary.

-> Need a "randomizer" for the wage. See Lecture 2c (slide 13ff), PS 1.2, or taxes & transfer programs below...

Measurement error

Measurement error in wages and division bias:

- For many workers, the wage rate is computed as earnings divided by hours, this generates a spurious negative correlation in hours, called the division bias (Borjas, 1980).
- Borjas (1980) considers a standard labour supply equation

$$\ln H_i^* = \alpha + \beta_1 \ln W_i^* + e_i$$

  where $W_i^* = E_i / H_i^*$ but we measure $H_i = H_i^* u_i$ with u_i *iid*.
- So the regression we have to run is

$$\ln H_i = \alpha + \beta_1 \ln W_i + e_i$$

Measurement error

- Borjas showed that the spurious negative correlation between H_i and W_i biases the elasticity estimate downward.
- Solution: when "usual hours last year" is the dependent variable, instrument average hourly earnings (earnings last year/usual hours last year) with alternative measure of average hourly earning (earnings last year/hours last week).
- What condition does the alternative measure of hourly earnings have to satisfy? Can it be measured with error?

Method: how IV solves the classical measurement error problem

Think of a generic bivariate regression

$$Y_i = \alpha + \beta X_i^* + e_i$$

and we observe

$$X_i = X_i^* + w_i.$$

The IV estimator is "reduced form over first stage" (again, Angrist and Pischke, 2008):

$$\begin{aligned} \lambda &= \frac{Cov(Y_i, Z_i)}{Cov(X_i, Z_i)} = \frac{Cov\left(\alpha + \beta X_i^* + e_i, Z_i\right)}{Cov(X_i, Z_i)} \\ &= \frac{\beta Cov\left(X_i^*, Z_i\right) + Cov(e_i, Z_i)}{Cov\left(X_i^*, Z_i\right) + Cov(w_i, Z_i)} \end{aligned}$$

Method: how IV solves the classical measurement error problem

$$\lambda = \frac{\beta Cov\left(X_i^*, Z_i\right) + Cov\left(e_i, Z_i\right)}{Cov\left(X_i^*, Z_i\right) + Cov\left(w_i, Z_i\right)}$$

so an instrument satisfying

$$Cov\left(w_i, Z_i\right) = Cov\left(e_i, Z_i\right) = 0$$

will solve the measurement error problem:

$$\lambda = \beta \frac{Cov\left(X_i^*, Z_i\right)}{Cov\left(X_i^*, Z_i\right)} = \beta$$

Method: how IV solves the measurement error problem

Why would an instrument satisfy

$$Cov\left(w_i, Z_i\right) = Cov\left(e_i, Z_i\right) = 0?$$

- We choose an instrument specifically to solve the measurement error problem.
  - For example, a second independent report related to the true regressor X_i^*.
  - "hours last week" as instrument for "usual hours last year".
- Question: does measurement error in the outcome variable Y_i (ln H_i in our application) still matter with the instrument?

Econometric Issues

TABLE 3
WAGE ELASTICITIES, STRAIGHT-TIME SAMPLE

	Dependent = $\ln(H_u)$		Dependent = $\ln(H_w)$	
Step	$\ln(E/H_u)$	$\ln(E/H_w)$	$\ln(E/H_u)$	$\ln(E/H_w)$
1	.0132	.0458	.0463	−.0105
	(.95)	(3.38)	(3.05)	(−.70)
2	.0133	.0463	.0479	−.0089
	(.96)	(3.40)	(3.14)	(−.59)
3	−.0168	.0243	.0280	−.0422
	(−1.10)	(1.62)	(1.66)	(−2.55)
4	−.0307	.0140	.0096	−.0614
	(−1.97)	(.92)	(.56)	(−3.68)
5	−.0383	.0099	.0163	−.0743
	(−2.37)	(.63)	(.09)	(−4.30)
6	−.0201	.0307	.0127	−.0721
	(−1.18)	(1.86)	(.67)	(−3.98)

Note: Step 1 regresses hours on wages. Step 2 adds nonwage income. Step 3 adds time remaining in the labor force, years of experience, number of children, and whether job information refers to current or last (if not currently working) job. Step 4 adds health and marital status. Step 5 adds education. Step 6 adds 11 one-digit industry dummies.

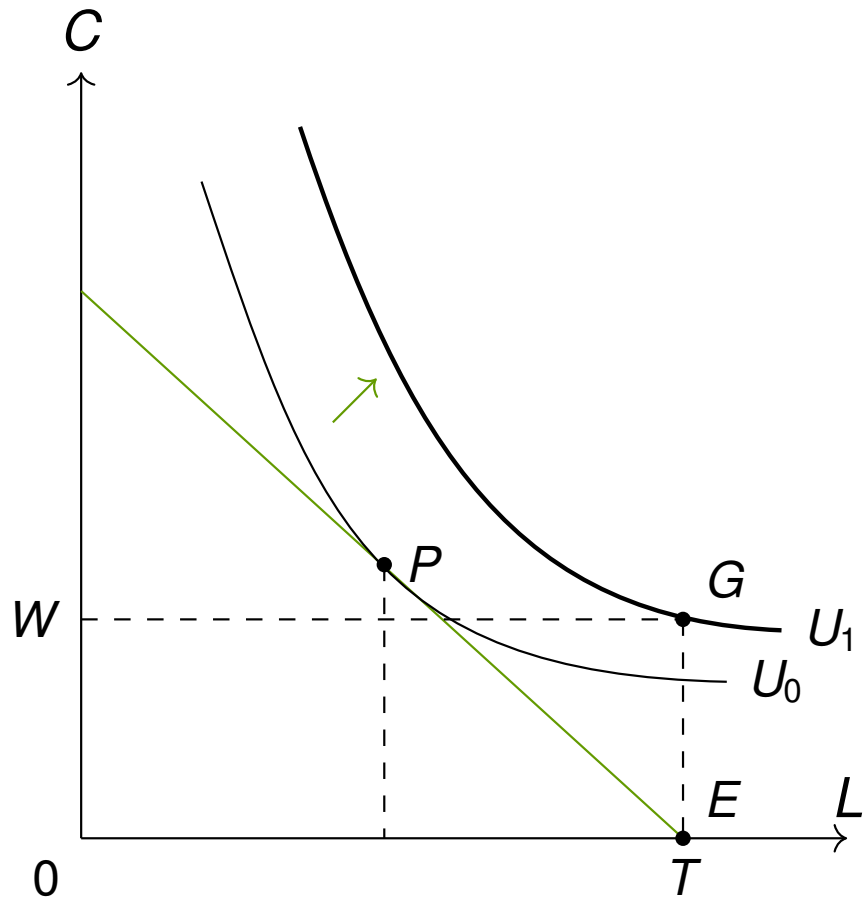
Source: Borjas (1980)

Using Taxes & Transfers to Estimate Labour Supply

Self-Sufficiency Project (SSP) as our example:

- SSP carried out in Canadian provinces of British Columbia (BC) and New Brunswick (NB) in 1990s.
  - Experimental sample randomly assigned between Nov 1992 and Feb 1995.
  - Sample size: program group 2859, control group 2827.
- SSP was designed to test a financial incentive to leave welfare – a policy to “make work pay”.
  - Targeted single parents on welfare for at least 12 months.
  - Intended effect on increasing long-term employment was through accumulated work experience.

Using Taxes & Transfers to Estimate Labour Supply

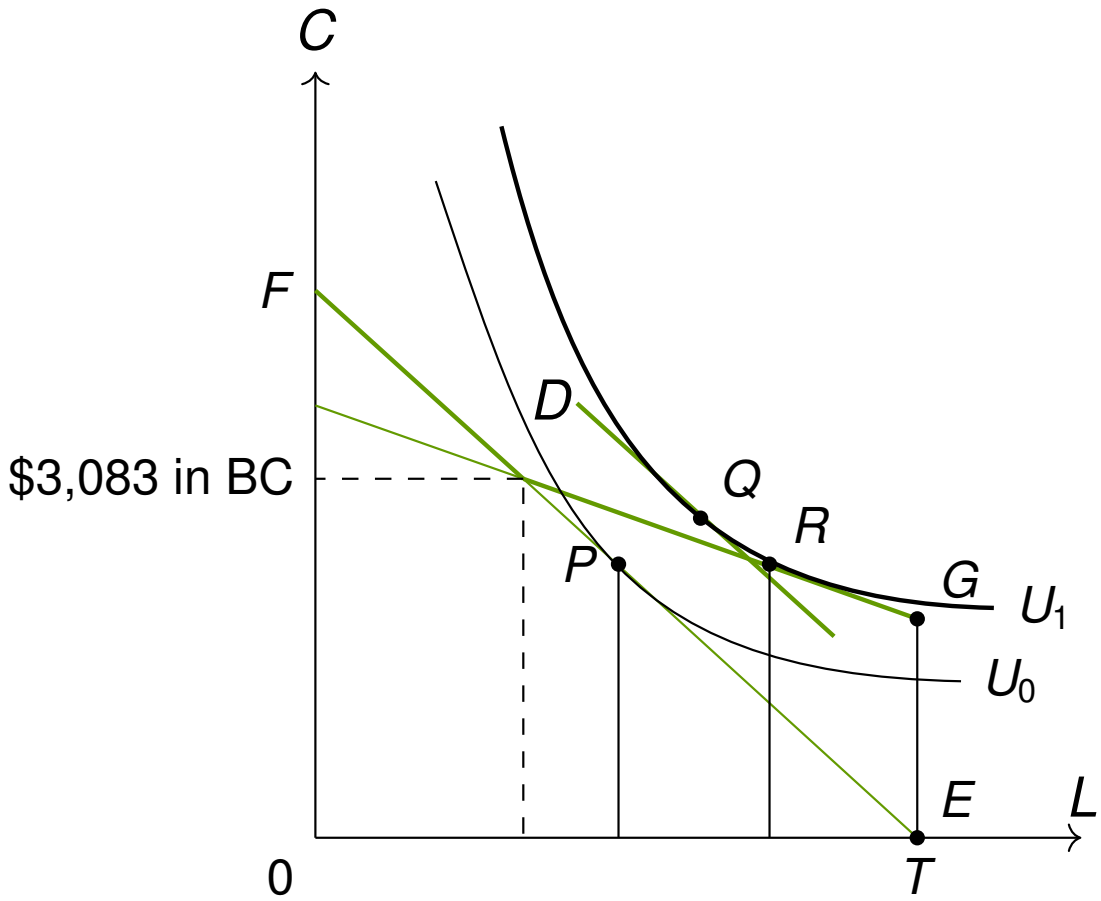


W is the welfare payment

SSP Earnings Supplement (ES)

- Supplement payment is ½ the difference between actual earnings and an earnings benchmark, equal to $2,500 per month in NB and $3,083 in BC.
- Payments not affected by unearned income or income of spouse/partner.
- Eligibility: 12 months following random assignment to obtain FT employment (FT = at least 30 hours per week on average over the month).
- Receipt: up to 3 years of ES providing that FT employment is maintained.
- Supplement quite generous relative to U.S. programs tested in early 1990s.

SSP Earnings Supplement (ES)



Under the program (point R) individuals work less than in P (no welfare system) but more than in G (with welfare system)

Descriptives and Balancedness Check

Source: Card and Hyslop (2005)

TABLE II

CHARACTERISTICS OF SSP EXPERIMENTAL SAMPLE[a]

			Program Group, by SSP Eligibility Status	
	Controls	Programs	Eligible	Ineligible
In British Columbia (%)	52.6	53.2	50.9	54.4
Male (%)	4.7	5.2	4.6	5.5
Mean age	31.9	31.9	31.1	32.4
Age 25 or less (%)	17.8	17.1	18.5	16.3
Never married (%)	48.1	48.3	48.0	48.5
Average number kids <6	0.7	0.7	0.7	0.7
Average number kids 6–15	0.8	0.8	0.8	0.8
Immigrant (%)	13.8	13.3	12.2	13.9
Grew up with two parents (%)	59.7	59.4	62.1	58.1
High school graduate (%)	44.6	45.7	56.9	39.9
Means years work exp.	7.4	7.3	8.6	6.7
Working at random assignment (%)	19.0	18.2	31.5	11.4
Months on IA last 3 years	29.6	30.1	29.2	30.6
IA continuously last 3 years (%)	41.5	43.8	36.3	47.7
Percent on IA by months since random assignment				
Month 6	90.8	83.1	62.8	93.5
Month 12	83.7	72.4	39.1	89.4
Month 18	77.9	65.9	27.2	85.6
Month 24	73.0	63.3	26.5	82.1
Month 36	65.4	58.8	27.6	74.8
Month 48	56.7	53.5	29.3	65.9
Month 60	50.6	48.4	28.5	58.5
Month 69	45.0	45.0	25.4	55.0
Number of observations	2,786	2,831	957	1,874

[a]Sample includes observations in the SSP Recipient Experiment who were on IA in the month of random assignment and the previous month. Eligible program group is the subset who received at least one SSP subsidy payment.

SSP effects (Card & Hyslop, 2005)

- Program effects estimated within a difference-in-differences (details on method later in the course) framework with controls because of differences between *T* and *C* in work experience and education upgrading.
- The effects on full-time employment and reduced welfare participation, although quite spectacular during the program, faded away after the end of the program.
  - At peak, SSP generated a 14 percentage point reduction in welfare participation.
- Given the requirements of the program of maintaining FT employment, there was a clearer positive impact on accumulated labor market experience.

SSP effects

Source: Card and Hyslop (2005)

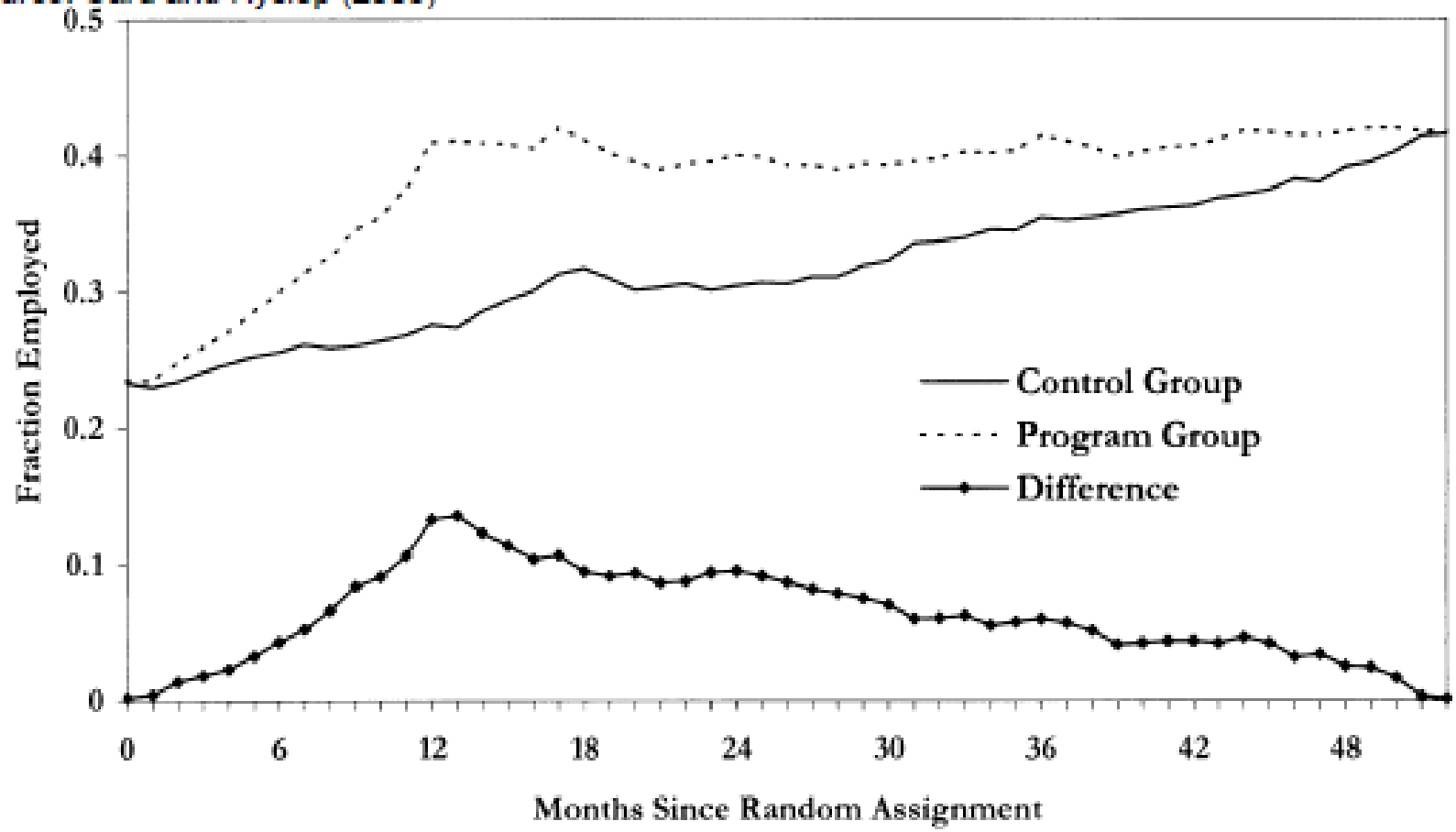


FIGURE 3.—Monthly employment rates.

SSP effects

In terms of wage growth, it turns out that the program requirement of maintaining full-time employment had some unintended consequences:

- "Establishment effect": In the short run, it forced some program participants to accept lower wages to get full-time employment rather than part-time. (Zabel et al, 2004).
- There is evidence of a relative wage progression of approximately 10 percentage points for the program participants who would not have found full-time employment in the first year without such an incentive.
- In the longer run, it prevented some program participants to upgrade their education yielding less wage growth than among the control group (Riddell and Riddell, 2006).

Using Taxes and Transfers to Estimate Labour Supply

Source: Card and Hyslop (2005)

TABLE III

SUMMARY OF LABOR MARKET OUTCOMES 52 MONTHS AFTER RANDOM ASSIGNMENT[a]

	Both Provinces	British Columbia	New Brunswick
Control group outcomes in month 52			
Percent employed	41.56	39.19	44.08
	(1.02)	(1.41)	(1.48)
Percent with reported wage	38.26	35.63	41.08
	(1.01)	(1.38)	(1.46)
Mean log hourly wage	2.17	2.36	1.99
	(0.01)	(0.02)	(0.02)
Cumulative employment since random assignment (in years)	1.41	1.33	1.49
	(0.03)	(0.04)	(0.05)
Program group outcomes in month 52			
Percent employed	41.69	37.73	46.05
	(1.00)	(1.36)	(1.47)
Percent with reported wage	39.45	35.04	44.31
	(0.99)	(1.34)	(1.47)
Mean log hourly wage	2.15	2.34	1.99
	(0.01)	(0.02)	(0.02)
Cumulative employment since random assignment (in years)	1.68	1.55	1.82
	(0.03)	(0.04)	(0.05)
Difference: Program group – Control group			
Percent employed	0.13	−1.46	1.97
	(1.43)	(1.96)	(2.08)
Percent with reported wage	1.19	−0.58	3.23
	(1.41)	(1.92)	(2.07)
Mean log hourly wage	−0.02	−0.02	−0.01
	(0.02)	(0.03)	(0.02)
Cumulative employment since random assignment (in years)	0.28	0.22	0.33
	(0.04)	(0.06)	(0.07)

Continues

the estimated marginal wage is roughly constant over the entire 52-month period suggests that any impact on wages of those who would have worked regardless of SSP is small.

[23]Wages for the labor market as a whole are 20–30% higher in Vancouver than in the areas

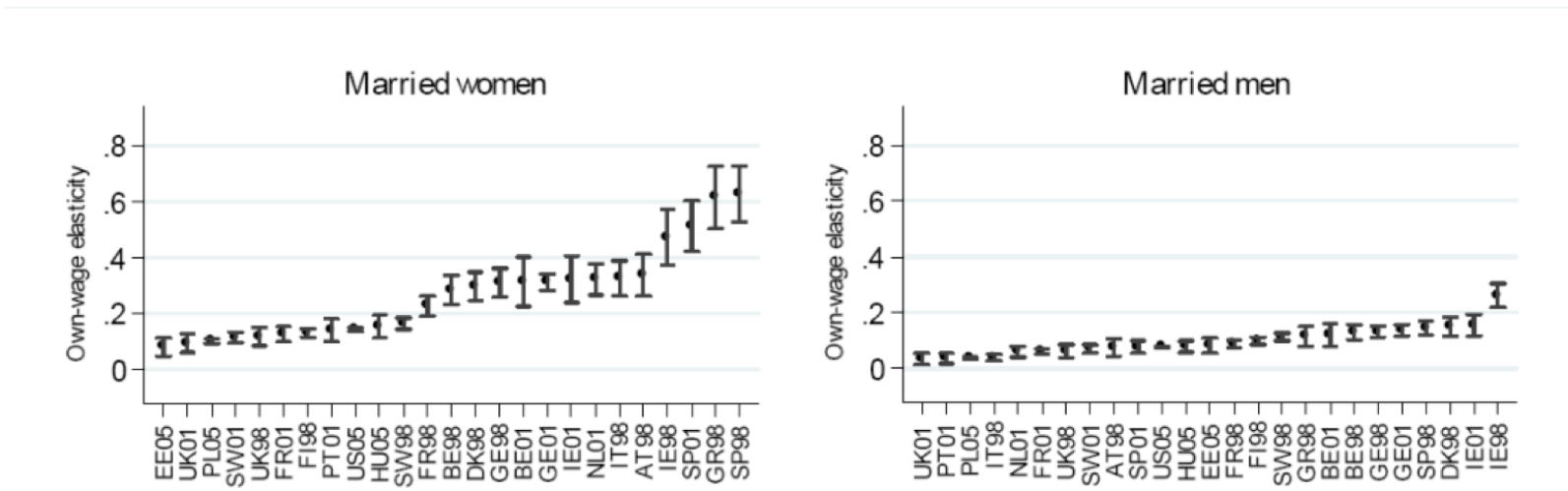
Findings in the Literature

There have been many studies estimating labour supply and income elasticities of labour supply, and there have been many meta-analysis of these studies.

- Evers, de Mooij and van Vuuren (2008) conclude that an uncompensated elasticity of 0.5 for women and 0.1 for men is a good reflection of what the literature reveals, although for the US it may be negative for men, due to the income effect.
  - For male workers, small wage effects
  - For female workers, much larger elasticities with larger variations across studies and declining over time as women have become more attached to the labour market

Findings in the Literature

- Bargain, Orsini and Peichl (2012) perform an extensive cross-country study of 17 European countries plus the US
  - the extensive (participation) margin dominates the intensive (hours) margin
  - for singles, this leads to larger labor supply responses in low-income groups
  - income elasticities are extremely small everywhere.



Take-Away from the Static Labour Supply Literature

1. Small elasticities for prime-age males
   - Probably institutional restrictions, need for at least one income, etc. prevent a short-run response
2. Larger responses for workers who are less attached to labor force
   - Married women, low incomes, retirees
3. Responses essentially driven by extensive margin
   - Extensive margin (participation) elasticity around 0.2-0.5
   - Intensive margin (hours) elasticity close to 0
4. Self-employed are much more responsive than employees, in particular the highest earners
   - Hours constraints likely important for employees

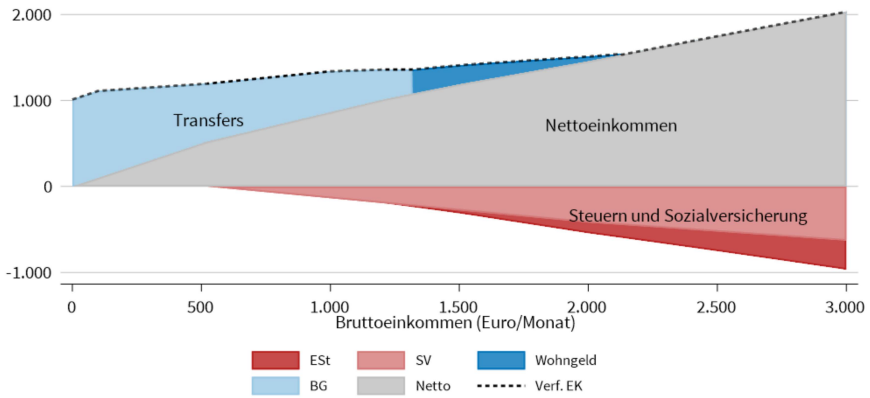
The Rise of European Un(der)employment

There are multiple hypotheses found in literature:

- Adverse shocks (Decline of labour demand)
  - Worldwide decrease of productivity since the 1970s
  - Higher energy prices after the 1970s oil-shocks
  - But: Note that these are worldwide occurring shocks. They were not limited to Europe.
- Inefficient labour market institutions (non-flexible wages, implicit minimum wages)
  - Costs for on-boarding and off-boarding
  - broad social security support (e.g. unemployment insurance)
    - USA: 20% - 30% of the wage for 6 months
    - Europe: 60% of the wage for 5 years or more.
  - But: Note that these institutions were implemented before 1960

So literature suggests a combination of both factors which are responsible for lower hours worked / employment in Europe.

Implicit Minimum Wage in Germany



Hinweis: Die Grafik zeigt den Verlauf des verfügbaren Einkommens (Verf. EK) bezogen auf das Bruttoeinkommen (Brutto) eines Haushalts nach Verrechnung aller Komponenten: Lohn- und Einkommensteuer (ESt), Sozialversicherungsbeiträge (SV), Wohngeld, Bürgergeld sowie Nettoeinkommen nach Abzug von Lohnsteuer und Sozialversicherungsbeiträgen (Netto). Alle Angaben in Euro/Monat. *Quelle:* ifo-MSM.

Source: Blömer, Maximilian, et al. "Die Ausgestaltung des Transferentzugs in der Interdependenz mit dem Bürgergeld, der Kindergrundsicherung und dem Wohngeld." ifo Forschungsberichte (2024)

- This model shows the amount of available income (dotted line) a single person household (entitled for unemployment benefit) has for different gross income.
- The rise of available income is flat for gross incomes between 0 to 2000 euros. So the effective marginal tax burden (Taxes + revocation of transfer payments for the next euro is very high).

Implicit Minimum Wage in Germany

Tabelle 25: Status quo (2023-7 + KGS) – EK-Komponenten – Single-Haushalt

	Brutto	0	520	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	7.000
−	ESt	0	0	0	34	123	227	338	456	582	716	858	1.580
−	Soli	0	0	0	0	0	0	0	0	0	0	0	14
−	SV (AN)	0	0	136	278	420	525	630	735	840	945	1.049	1.261
=	Netto	0	520	864	1.188	1.457	1.748	2.032	2.309	2.578	2.839	3.094	4.145
+	Wohngeld	0	0	0	222	55	0	0	0	0	0	0	0
+	BG inkl. KdU	1.012	676	476	0	0	0	0	0	0	0	0	0
=	verf. EK	1.012	1.196	1.340	1.410	1.512	1.748	2.032	2.309	2.578	2.839	3.094	4.145

Hinweise Tabellarisch wird eine Zerlegung des Haushaltseinkommens in einzelne Komponenten im Status quo (2023-7 + KGS) dargestellt. Alle Werte in Euro/Monat. Der zugrundeliegende Musterhaushalt ist ein Single-Haushalt. *Quelle:* ifo-MSM.

If we assume you're offered a 40 hour job for the minimum wage. So you would earn 2000 euros gross and 1512 euros after taxes. And if your harm from work or negative utility (or net wage rate in the black market) in euros is 5 euros per hour.

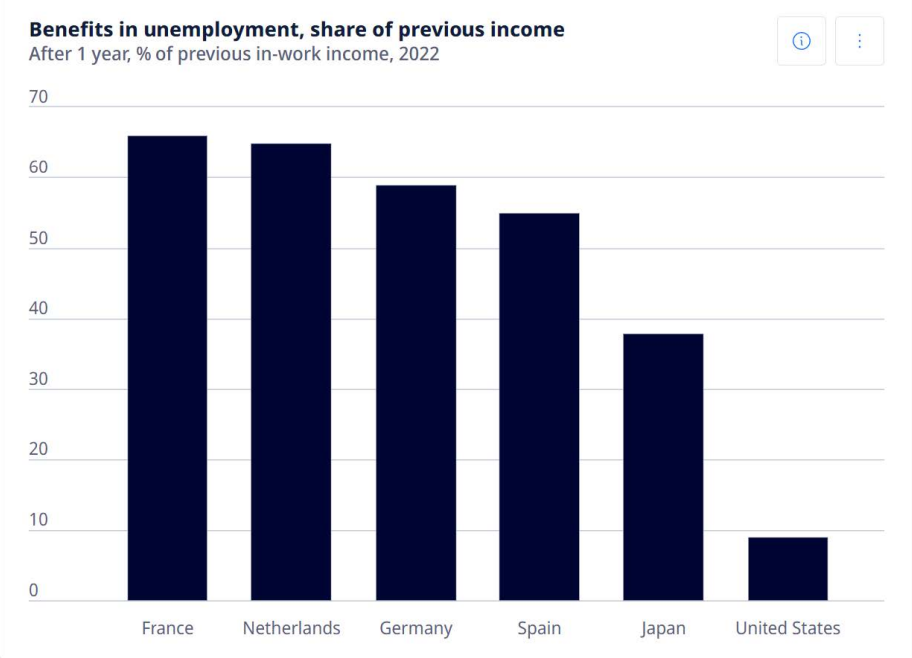
Would you take the job?

Implicit Minimum Wage in Germany

Would you take the job?

Relative to being unemployed, you would earn 3.12 euros (=1512-1012)/160) more per hour.
$\rightarrow$ Since this is lower than your harm from work, you would not work and therefore choose to be unemployed.
We assume, counterfactually, that with 0 euros gross income, your available income would be 500 euros. Then you would accept the job because you would earn 6.32 euros more per hour (= (1512 - 1012) / 160). $\rightarrow$ You are not unemployed.
The welfare system here acts as an implicit minimum wage, which creates (voluntary) unemployment or underemployment (too few hours).

Wage Replacement Benefits



Source: OECD

- People who become unemployed are often entitled to unemployment benefits (before being demoted to Bürgergeld).
- In Germany, unemployment benefits replace 60% of the net income for people without children and 67% for those with children.
- In Europe, unemployment benefits are generally higher than in Japan or the USA.
- Wage replacement benefits can also act as a minimum wage.

Labor Market Institutions and the Reduction of Working Hours

Another way institutions can influence the labor market is by affecting the average working hours in a country for full-time employment. Let's return to our example:

- Assume that harm from work is not a fixed number, but increases with the number of hours worked. For example, the harm for the first 20 hours of work is 3 euros per hour, and for all additional hours, it is 6 euros per hour.
- Now, let's assume you are offered a job at the minimum wage, but you can now choose how many hours you work. The gross-to-net income table from above still applies.
- You would choose to work 20 hours because for each additional hour, the extra income does not cover the additional harm.
- If the available income at 0 euros gross were only 500 euros, you would work 40 hours.

Labor Market Institutions and the Reduction of Working Hours

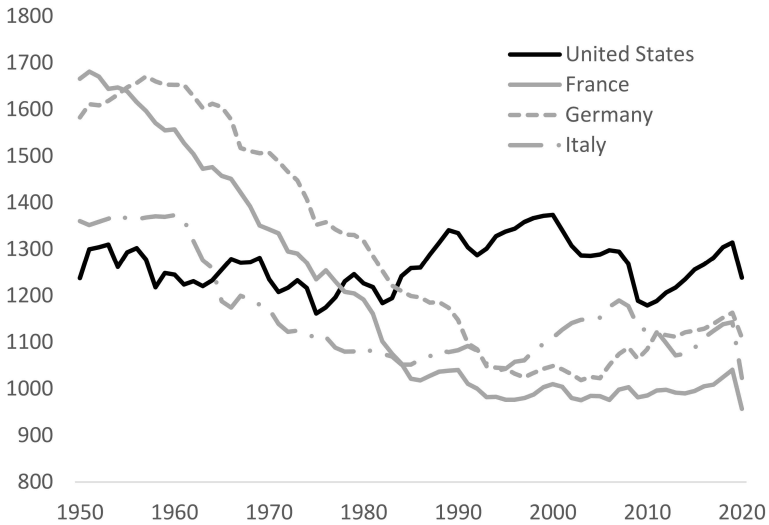


Figure 12: Annual Hours Worked per Working-Age Person

Note: This figure plots annual hours worked per person of age 15-65 in the United States, France, Germany, and Italy from 1950 to 2000. Data on total hours worked are from the Conference Board Total Economy Database. Data on the population aged 15-65 is from the OECD Employment and Labor Force Statistics database.

Source: Jon Steinsson: Work and Leisure

- A high effective marginal tax rate can lead to fewer working hours.
- This applies not only to low incomes but also to higher incomes that may be subject to increasing marginal taxation.
- Empirically, Europeans work significantly fewer hours than Americans. Some estimates suggest that about 50% of the difference in working hours can be explained by taxes.

Additional Hypotheses on Differences in Working Hours

- European culture: However, in the 1960s, more hours were worked in Europe than in the USA.
- Coordination problems in the USA: Workers might want to work less if other workers also worked less. One possible reason is competition between workers. Unions in Europe may have solved this coordination problem.
- Regulations, such as maximum working hours per day or sick leave. For example, in Germany
  - very generous salary continuation / replacement as well as easy to report sick.
  - among the highest numbers of sick days in the OECD (25 and four times the number in the US or UK).

Basic Readings

- Angrist, Joshua D., and Jörn-Steffen Pischke. Mostly harmless econometrics: An empiricist's companion. Princeton university press, 2009.
- Borjas, George. *Labor Economics*. Chapter 2.