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# Lecture 2a:
# The Theory of Labour Supply

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Empirical Economics

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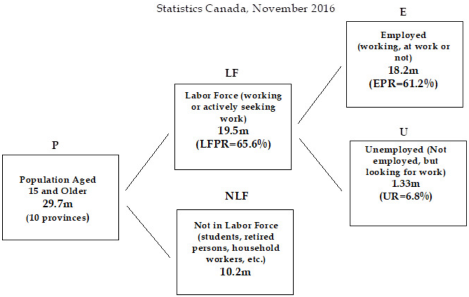
# Outline

1. Basic Trends and Stylized Facts
2. Static Model
   a. Decision of whether to work or not: Extensive margin
   b. Decision of how many hours to work: Intensive margin
3. Comparative Statics
4. Estimation Equation

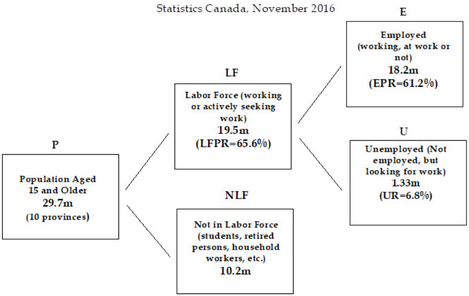
# Basic trends and stylized facts




Labour Force Concepts:

- Labour Force = Employed + Unemployed
  - $LF = E + U$
  - Size of LF does not tell us about "intensity" of work
- Labour Force Participation Rate
  - $LFPR = LF/P$
  - $P$ is the civilian adult population 15 years or older not in institutions

# Basic trends and stylized facts




Labour Force Concepts:

- Employment to Population Ratio (percent of population that is employed)
  - $EPR = E/P$
  - Employed at work and not at work (e.g. maternity or sick leave) sometimes distinguished
- Unemployment Rate
  - $UR = U/LF$

# Basic trends and stylized facts

**TABLE 2-3 Labor Supply in the United States, 2017 (Persons Aged 25–64)**

Source: U.S. Bureau of Labor Statistics, *Current Population Survey,* Annual Social and Economic Supplement, March 2017. The average number of hours worked is calculated in the subsample of workers. The percent of workers in part-time jobs refers to the proportion working fewer than 30 hours per week.

| | Labor Force Participation Rate | | Annual Hours of Work | | Percent of Workers in Part-Time Jobs | |
|---|---|---|---|---|---|---|
| | Men | Women | Men | Women | Men | Women |
| All persons | 83.1 | 71.4 | 2,170 | 1,933 | 4.3 | 12.9 |
| Educational attainment: | | | | | | |
| Less than 12 years | 72.1 | 45.6 | 2,033 | 1,753 | 5.4 | 19.7 |
| 12 years | 79.1 | 63.3 | 2,124 | 1,875 | 4.7 | 14.1 |
| 13–15 years | 82.5 | 73.5 | 2,166 | 1,906 | 4.8 | 13.4 |
| 16 years or more | 90.4 | 80.5 | 2,235 | 2,000 | 3.4 | 11.2 |
| Age: | | | | | | |
| 25–34 | 87.1 | 75.6 | 2,101 | 1,904 | 5.7 | 12.0 |
| 35–44 | 89.2 | 75.1 | 2,201 | 1,928 | 2.7 | 13.0 |
| 45–54 | 85.3 | 74.7 | 2,221 | 1,978 | 2.9 | 12.0 |
| 55–64 | 70.5 | 60.0 | 2,160 | 1,922 | 6.2 | 15.2 |
| Race: | | | | | | |
| White | 83.8 | 73.1 | 2,208 | 1,933 | 4.1 | 13.8 |
| Black | 74.9 | 72.0 | 2,096 | 1,963 | 6.0 | 9.6 |
| Hispanic | 85.6 | 64.7 | 2,086 | 1,882 | 4.0 | 12.7 |
| Asian | 87.5 | 68.2 | 2,121 | 1,961 | 3.1 | 11.3 |

# Static Model

- In neo-classical theory, the individuals' decisions of whether or not participate in the labour market and of how many hours to work each week (and weeks per year) are modeled in static framework of consumption-leisure choice.
- From a policy point of view, this model has been very important to evaluate the potentially negative effects on labour supply of tax and transfer programs.
- From an econometric viewpoint, the analysis will provide us with a classic example of correction for measurement error, selection and omitted variables biases.

# Static Model

- Estimation of “the” elasticity of labour supply $\frac{\%\Delta h}{\%\Delta w}$ has long been an important quest for labour econometricians
- The more modern approaches have emphasized clean sources of identification coming from natural and quasi-natural experiments, as well as field experiments.

# Static Model

- The standard static, within-period labour supply model is an application of the consumer's utility maximization problem over consumption and leisure.
- Assume that each individual has a quasi-concave utility function:

$$U(C, L)$$

  where $C$ and $L$ are within-period consumption and leisure hours.
- Then utility is assumed to be maximized subject to the budget constraint

$$pC + wL = Y + wT$$

  where $w$ is the hourly wage rate, $Y$ non-labour income, and $T = H + L$ total time available, $H$ the number of hours of work.

# Static Model

- Partial equilibrium: $p$ and $w$ exogenously given. In fact, normalize such that consumption good is the numeraire $p = 1$.
  - Everything relative to units of consumption good.

- $M = Y + wT$ is sometimes called full income.
- $H(L)$, $C$ are endogenous in this model.
- $T$ and $Y$ are also exogenous in this model.
- The consumer may choose his/her hours of work $H(L)$ by selecting across employers offering different packages of hours of work and wages.

# Static Model

Preferences satisfy standard conditions:

$$U_C > 0 \quad U_L > 0 \quad U_{CC} < 0 \quad U_{LL} < 0$$

- quasi-concave utility function (“non-decreasing”)⇒convex indifference curves

The consumer / worker maximizes utility:

$$max_{C,L} U(C, L) \quad s.t. \quad C + wL \leq M \quad \& \quad C, L \geq 0$$

- interior solution or corner solution.

# Static Model–Interior Solution

- Lagrangian:

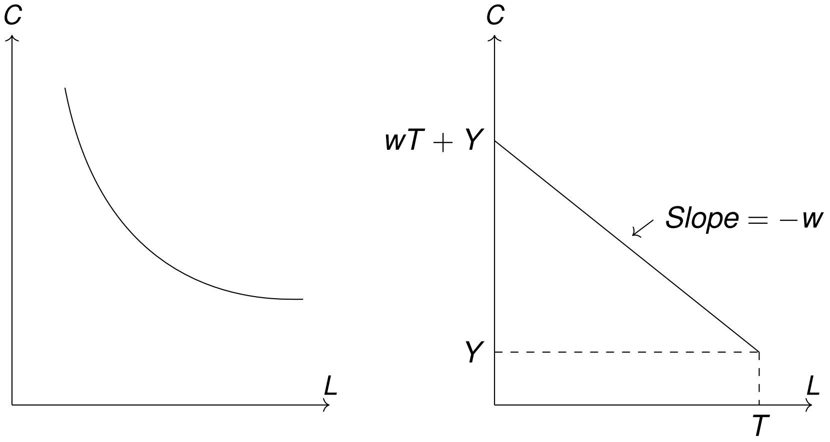
$$\mathcal{L}(C, L, \mu) = U(C, L) - \mu \cdot (C + wL - M)$$

- First-order conditions:

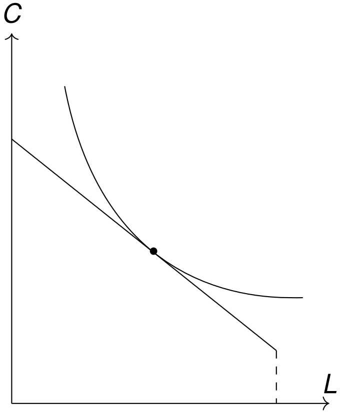
$$U_C(C, L) = \mu \qquad U_L(C, L) = \mu w$$

- Complementary slackness condition (Kuhn-Tucker): $\mu \cdot (C + wL - M) = 0$ with $\mu > 0$
- Economic interpretation of Lagrange Multiplier as shadow price $\mu = \frac{\partial U(C^*, L^*)}{\partial M}$.

# Static Model



# Static Model—Interior Solution




For the labour market participant, the indifference curve is tangent to the budget line at the optimum point (slope $= -w$).

# Static Model

First Order Conditions (FOC):

- In the case of an interior solution, the individual chooses to participate in the labour market $L^* < T$, the first-order conditions equate the marginal rate of substitution ($MRS_{LC}$) to the real wage rate

$$\frac{U_L(C,L)}{U_C(C,L)} \mid_{L^*,C^*} = w$$

- It is important to distinguish the characteristics of the interior solution for hours of work, $H > 0, (L < T)$ from the corner solution, $H = 0, (L = T)$.

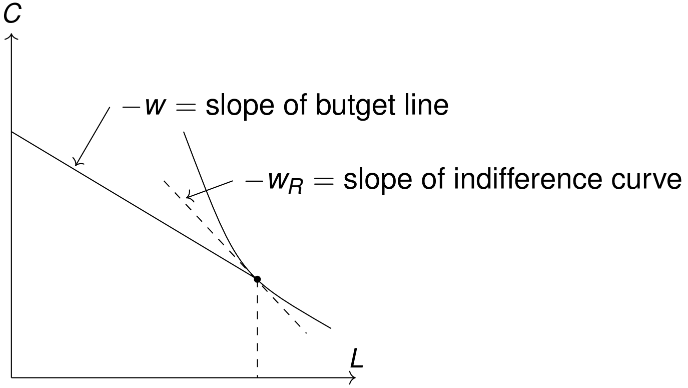
# Static Model

- In the case of the corner solution, $L^* = T$,

$$w \leq w_R = \frac{U_L(C, L)}{U_C(C, L)} \mid_{L^*=T, C^*}$$

where the reservation wage, $w_R$, is equal to the negative of $MRS_{HC}$ of working hours for commodities at $H = 0, (L = T)$.

# Static Model—Corner Solution




For the non-participant, the reservation wage is the slope of the indifference curve at the optimum point.

(note: slope $= w$ might differ for workers with different skills...)

# Comparative Statics

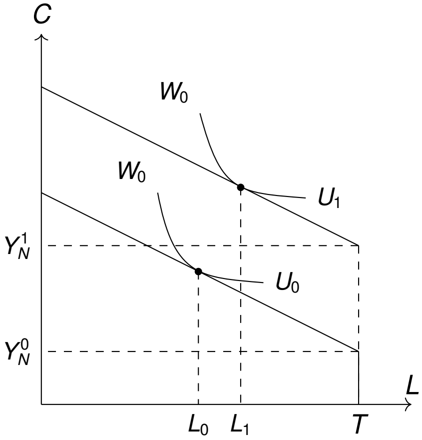
Solving the above first order conditions (FOCs) yields the Marshallian (or uncompensated) demand functions for goods $C^* = C(w, Y)$ and leisure $L^* = L(w, Y)$ or equivalently the labour supply function

$$H^* = H(w, Y)$$

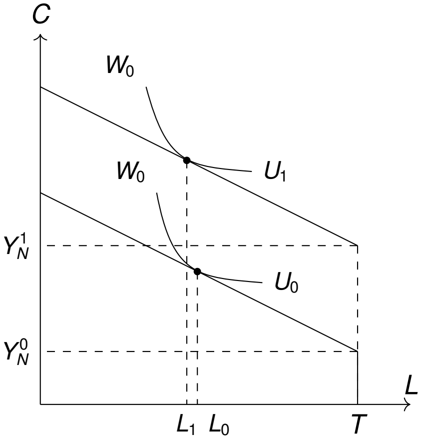
- The comparative statics of the impact of changes in income, $\frac{\partial H}{\partial Y}$, and wage rate $\frac{\partial H}{\partial w}$, of the labour supply function $H^* = H(w, Y)$ are best illustrated in a diagram of consumption-leisure choice.
- The general effects are the following.

# Comparative Statics (non-labour income $Y$)

Leisure as a normal good



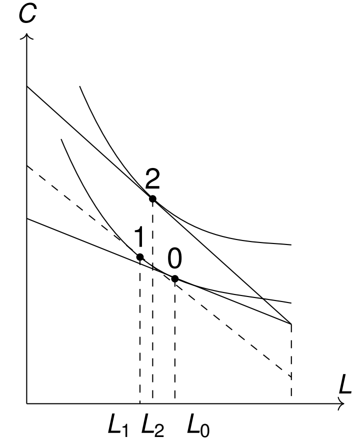

Leisure as a inferior good



# Comparative Statics (non-labour income $Y$)

- An increase in non-labour income: An increase in non-labour income will shift the budget line outwards without changing the slope of the line: this is a pure income effect.
- The effect on the optimal amount of leisure consumed or hours worked can then be summarized as:
  - $L$ will rise and $H$ will fall if leisure is a normal good.
  - $L$ will fall and $H$ will rise if leisure is an inferior good.
- There are strong reasons to believe that leisure is a normal good, e.g. those who win the lottery (a large increase in non-labour income) are more likely to work less afterwards.

# Comparative Statics (wage rate $w$)




Moving from $L_0$ to $L_1$ is the substitution effect and from $L_1$ to $L_2$ the income effect.

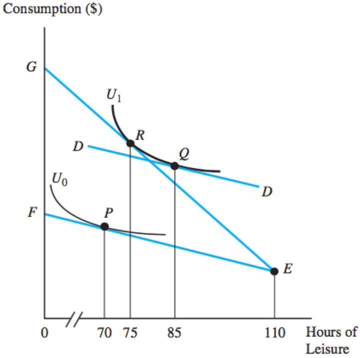
# Comparative Statics (wage rate *w*)

- An increase in the real hourly wage: An increase in the real hourly wage will pivot the budget line about the point where L=T making the line steeper.
- Here there are two effects:
  - An **income effect**. Individuals are better-off than before so there is a positive income effect that, because leisure is a normal good, makes individuals work fewer hours than before.
  - A **substitution effect**. An hour of work now buys more consumption than previously so that there is an incentive to increase consumption and reduce leisure. Hours of work will rise as a result.

# Comparative Statics (wage rate $w$)

**FIGURE 2-9 Decomposing the Impact of a Wage Change into Income and Substitution Effects**

An increase in the wage rate generates both income and substitution effects. The income effect (the move from point $P$ to point $Q$) reduces hours of work; the substitution effect (the move from $Q$ to $R$) increases hours of work.




(*a*) Income Effect Dominates

(*b*) Substitution Effect Dominates

Source: Borjas (2015)

# Comparative Statics (wage rate *w*)

- Hence, the impact of a change in the wage on hours of work is theoretically ambiguous. They may rise or fall.
- There is one exception to this: for non-participants there is no income effect (at the margin) as they have no labour income. Also nobody can be induced to reduce hours of work to zero as a result of an increase in the wage.
- Empirical policy evaluations often in this area (EITC, programs incentivizing single mothers to take up employment).
- **Exercise:** Policy questions in Germany
  - What are the effects of abolishing “Bürgergeld”?
  - What about a 1000eur bonus for picking up work (and work at least 30 hours)?

# Comparative Statics

How can we quantify these effects?

- The Hicksian (or compensated) labour supply function $H^C(w, \overline{U})$ is the solution to the expenditure minimization problem

$$E(w, \overline{U}) = \min(C - wH) \text{ subject to } U(C, H) \geq \overline{U}$$

and corresponds to the following uncompensated labour supply function:

$$H(w, \overline{Y}) = H^C(w, \overline{U}) \text{ where } \overline{Y} = E(w, \overline{U})$$

- Differentiating with respect to $w$ and applying the chain-rule

$$\frac{\partial H^C}{\partial w} |_{\overline{U}} = \frac{\partial H}{\partial w} + \frac{\partial H}{\partial Y}\frac{\partial E}{\partial w}$$

# Comparative Statics

- With the application of Sheppard's Lemma ($\frac{\partial E}{\partial w} = -H$ in optimum), we get the Slutsky equation

$$\frac{\partial H}{\partial w} = \underbrace{\frac{\partial H^C}{\partial w} \mid_{\overline{U}}}_{\text{Subs.Effect}} + \underbrace{H\frac{\partial H}{\partial Y}}_{\text{Inc.Effect}}$$

  where the effect of the wage change is decomposed into a substitution effect plus an income effect.
- NB: Sheppard's Lemma is an application of the *envelope theorem*, i.e., that marginal price changes *only have direct effects* on the objective function in optimum.

# Comparative Statics

- Multiplying the entire equation by $\frac{w}{H}$ and the last term (income effect) by $\frac{Y}{Y}$

$$\frac{\partial H}{\partial w}\frac{w}{H} = \frac{\partial H^C}{\partial w}\ |_{\overline{U}} \cdot \frac{w}{H} + \frac{wH}{Y}\frac{\partial H}{\partial Y}\frac{Y}{H}$$

or in terms of elasticites

$$\varepsilon_{Hw} = \varepsilon^C_{Hw} + s_L \eta_{HY}$$

# Comparative Statics

Thus, there are three "sufficient statistics" of labour supply

- the uncompensated wage elasticity: the % change in labour supply resulting from 1% change in the wage rate; sign is theoretically ambiguous as the positive substitution effect can sometimes be dominated by the negative income effect
  - $\varepsilon_{Hw} = \frac{\partial H}{\partial w}\frac{w}{H} > 0 (< 0)?$
- the compensated wage elasticity: the % change in labour supply resulting from 1% change in the wage rate, after compensation for the wage change; sign is positive as it reflects a pure substitution effect
  - $\varepsilon_{Hw}^{C} = \frac{\partial H^{C}}{\partial w}\frac{w}{H} > 0$

# Comparative Statics

- the income elasticity: the % change in labour supply resulting from 1% change in non-labor income; sign is expected to be negative
  - $\eta_{HY} = \frac{\partial H}{\partial Y} \frac{Y}{H} < 0$
- the income elasticity's effect is larger the higher share of income is from labour $s_L \frac{wH}{Y}$
  - as we said before, no income effect if the person is not working ($s_L = 0$).

# Comparative Statics

- The simple consumption-leisure model can be extended to analyze labour supply under various conditions:
  1. introducing the fixed (money) cost of working or time cost (commuting) of working
  2. moonlighting (second job) and overtime pay
  3. should a firm offer flexible hours (part-time) or hire only full-time workers?
  4. family labour supply (actually more than a simple extension)

**Exercise:** Can you construct extensions 1 and 2?

# Estimation Equation

- Suppose that we have individual data on hours of work $H_i$, the wage rate $w_i$, and on non-labor income $Y_i$, we could estimate a simple OLS regression

$$H_i = \beta_0 + \beta_1 w_i + \beta_2 Y_i + \varepsilon_i$$

# Estimation Equation

- Then the estimated effects:
  - $\hat{\beta}_1$ will be the overall (uncompensated) effect, $\frac{\partial H}{\partial w}$
  - $\hat{\beta}_2 \overline{H}$ will be the income effect, $\overline{H}\frac{\partial H}{\partial Y}$, (evaluated at mean hours)
  - $\hat{\beta}_1 - \hat{\beta}_2 \overline{H}$ will be the substitution effect, $\frac{\partial H}{\partial w} - \overline{H}\frac{\partial H}{\partial Y}$, (evaluated at mean hours)
  - $\hat{\beta}_2 \frac{\overline{Y}}{\overline{H}}$ will be the income elasticity of labour supply, $\frac{\partial H}{\partial Y}\frac{Y}{H}$, (evaluated at mean hours and mean income)

# Basic Readings

- Borjas, George. *Labor Economics*. Chapter 2.
- Borjas, George. *Labor Economics*. Mathematical Appendix, Sections 1–2.